CHAPTER

7

Interest, Time Value of Money, Taxes, and Fixed Charges

As noted earlier, there are many economic parameters besides capital investment and operating expenses that can have an effect on the decision of whether to appropriate funds for a proposed project. This is particularly true when the required funds for the project may have to be borrowed. If such is the case, there will be an interest charge for the use of the required funds. If internal funds are available, a decision must be made between the use of the funds for the proposed project or for some other, more profitable project. In the decision-making process, a careful analysis of the time value of money will help establish the worth of earnings and investments.

Consideration must also be given to the effect of taxes on the net profit of the proposed project. For example, income taxes presently range from 15 to 39 percent of the taxable income, and such a tax may have a major impact on the net, after-taxes, earnings of a project. In addition to income taxes, there are other fixed charges such as property taxes, depreciation, and insurance that, once the proposed assets have been acquired, continue no matter what level of business is maintained.

This chapter examines the various forms of interest available to the borrower or the lender. It identifies various means of assessing the worth of earnings and investments that can provide meaningful economic comparisons between various investment opportunities. Finally, the chapter provides a summary of the taxes and fixed charges that need to be considered in the preparation of the economic assessment of the proposed project.

INTEREST

Interest is the cost of borrowed money, or the earnings on money loaned. *Principal* refers to the original amount or the remaining unpaid amount of a loan. *Interest rate* is defined as the amount of money earned by, or paid on, a unit of principal in a unit of time, expressed as a fraction or percentage per year. Interest paid is an expense of business operation that must be included in the analysis of business profitability.

Simple Interest

The simplest form of interest requires compensation payment at a constant interest rate based only on the original principal. Thus, if \$1000 were loaned for a total time for 4 years at a constant interest rate of 10 percent per year, the simple interest earned would be

$$(\$1000)(0.1)(4) = \$400$$

If P represents the principal, N the number of time units or interest periods, and i the interest rate based on the length of one interest period, the amount of simple interest I accumulated during N interest periods is

$$I = PiN \tag{7-1}$$

The principal must be repaid eventually; therefore, the entire amount of principal plus simple interest due after N interest periods is

$$F = P + I = P(1 + iN) \tag{7-2}$$

where F is the total amount of principal and accumulated interest at time N.

Compound Interest

Compound interest is almost universally used in business transactions. *Compound interest* is the interest earned on accumulated, reinvested interest as well as the principal amount. This applies to payments on loans or interest on investments. Thus, an investment of \$1000 at an interest rate of 10 percent per year payable annually would earn \$100 in the first year. If no principal were removed and the interest were left in the investment to earn at the same rate, then at the end of the second year $(\$1000 + \$100)(0.10) = \$110$ in interest would be earned and the total *compound amount* would be

$$\$1000 + \$100 + \$110 = \$1210$$

The compound amount earned after any discrete number of interest periods can be determined as follows:

Period	Principal at start of period	Interest earned during period (i = interest rate based on length of one period)	Compound amount F at end of period
1	P	Pi	$P + Pi = P(1+i)$
2	$P(1+i)$	$P(1+i)(i)$	$P(1+i) + P(1+i)(i) = P(1+i)^2$
3	$P(1+i)^2$	$P(1+i)^2(i)$	$P(1+i)^2 + P(1+i)^2(i) = P(1+i)^3$
N	$P(1+i)^{N-1}$	$P(1+i)^{N-1}(i)$	$P(1+i)^N$

Therefore, the total amount of principal plus interest earned after N interest periods is

$$F = P(1+i)^N \tag{7-3}$$

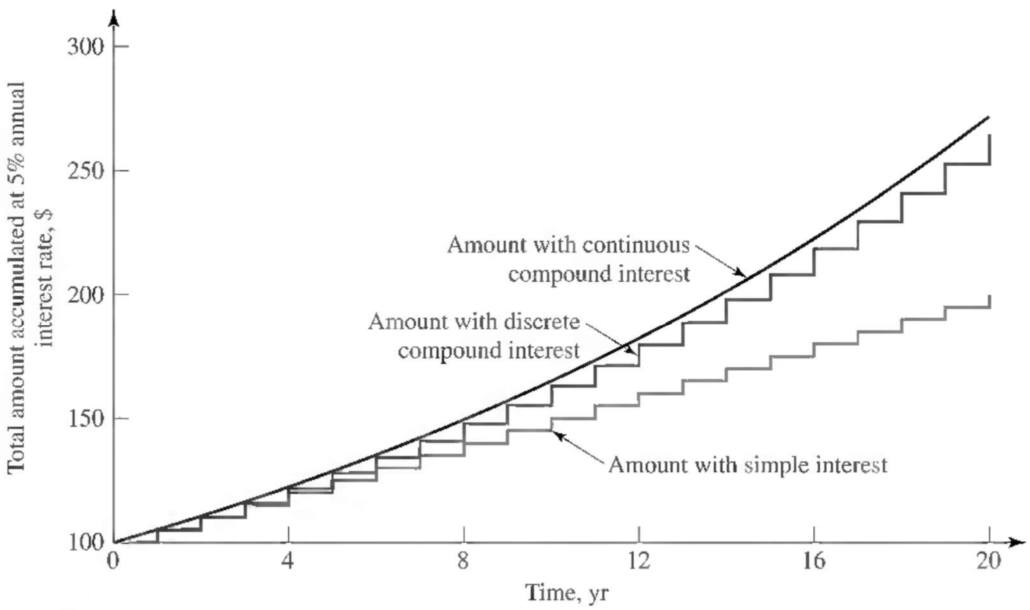

Figure 7-1
Comparison among total amounts accumulated with simple annual interest, discrete annually compounded interest, and continuously compounded interest

The term $(1+i)^N$ is commonly referred to as the *discrete single-payment compound amount* factor, or the *discrete single-payment future-worth* factor. It is represented by the functional form $(F/P,\ i,\ N)$,[†] and

$$(F/P,\ i,\ N) = (1+i)^N \tag{7-4}$$

Equation (7-3) can also be written as

$$F = P(F/P,\ i,\ N) \tag{7-5}$$

The factor value is easily calculated from Eq. (7-4).

Figure 7-1 shows a comparison among the total amounts earned at different times for cases using simple interest, discrete annually compounded interest, and continuously compounded interest, as described later in this chapter.

Nominal and Effective Interest Rates

In common industrial practice, the length of the discrete interest period is taken to be 1 year, and the fixed interest rate i is based on 1 year. However, there are cases where other time periods are employed. Even though the actual interest period is not 1 year, the interest rate is often expressed on an annual basis. Consider an example in which the interest rate is 3 percent per period and the interest is compounded at half-year periods. A rate of this type is referred to as *6 percent per year compounded semiannually,*

[†]This notation is from the American National Standards Institute publication ANSI Z94.7-2000. It can be read as: multiply P by the factor value to obtain F, for interest rate i and time period N. It is, as the symbol implies, the ratio of F to P for a given i and N.

or just *6 percent compounded semiannually* for simplification. An interest rate stated as an annual rate but compounded other than annually is designated a *nominal interest rate*. The actual annual return (the *effective interest rate*) would not be 6 percent, but would be higher because of the effect of compounding twice per year. Whenever no period for interest is stated or obvious, assume it to be per year.

The effective interest rate i_{eff} is the rate which, when compounded once per year, gives the same amount of money at the end of 1 year, as does the nominal rate r compounded m times per year. The interest rate per period is r/m, and the amount at the end of the year, using Eq. (7-3), is

$$F = \left(1 + \frac{r}{m}\right)^m \tag{7-6}$$

The amount given by compounding at the effective rate i_{eff} at the end of 1 year is

$$F = (1 + i_{\text{eff}})^1 \tag{7-7}$$

These two values of F must be the same, given the definitions of nominal and effective interest; therefore,

$$1 + i_{\text{eff}} = \left(1 + \frac{r}{m}\right)^m \tag{7-8}$$

The effective annual interest rate can be determined from this relation as

$$i_{\text{eff}} = \left(1 + \frac{r}{m}\right)^m - 1 \tag{7-9}$$

Thus, if $r = 0.06$ and $m = 2$ (twice on an annual basis), then

$$i_{\text{eff}} = \left(1 + \frac{0.06}{2}\right)^2 - 1 = 1.0609 - 1 = 0.0609$$

Nominal interest rates should always include a qualifying statement indicating the compounding period. For example, using the common annual basis, \$100 invested at a nominal interest rate of 20 percent compounded annually would amount to \$120.00 after 1 year; if compounded semiannually, the amount would be \$121.00; and, if compounded continuously, the amount would be \$122.14. The corresponding effective interest rates are 20.00, 21.00, and 22.14 percent, respectively.

EXAMPLE 7-1 Applications of Different Types of Interest

It is desired to borrow \$1000 to meet a financial obligation. This money can be borrowed from a loan agency at a monthly interest rate of 2 percent. Determine the following:

a. The total amount of principal plus simple interest due after 2 years if no intermediate payments are made.

b. The total amount of principal plus compounded interest due after 2 years if no intermediate payments are made.

c. The nominal interest rate when the interest is compounded monthly.

d. The effective interest rate when the interest is compounded monthly.

■ **Solution**

a. The length of one interest period is 1 month, and the number of interest periods in 2 years is 24. For simple interest, the total amount due after n periods at a periodic (in this case monthly) interest rate of i is given by Eq. (7-2):

$$\begin{aligned} F &= P(1 + iN) \\ &= 1000[1 + (0.02)(24)] = \$1480 \end{aligned}$$

b. For compound interest, the total amount due after N periods at a periodic interest rate i is obtained from Eq. (7-3):

$$\begin{aligned} F &= P(1 + i)^N \\ &= \$1000(1 + 0.02)^{24} = \$1608 \end{aligned}$$

c. Nominal interest rate is (2)(12), or 24 percent, per year compounded monthly.

d. Use Eq. (7-9) where $m = 12$ and $r = 0.24$.

$$\begin{aligned} i_{\text{eff}} &= \left(1 + \frac{r}{m}\right)^m - 1 \\ &= \left(1 + \frac{0.24}{12}\right)^{12} - 1 = 0.268 = 26.8\% \text{ per year} \end{aligned}$$

■

Continuous Interest

The preceding discussion of simple and compound interest considered only the form of interest in which the payments are charged at periodic and discrete intervals, where the intervals represent a finite length of time with interest accumulating in a discrete amount at the end of each interest period. Although in practice the basic time interval for interest accumulation is usually taken as 1 year, shorter periods can be used, such as 1 month, 1 day, 1 h, or 1 s. The extreme case, of course, occurs when the time interval becomes infinitesimally small so that the *interest is compounded continuously*. The concept of continuous interest is that the cost or income due to interest flows regularly, and this is just as reasonable an assumption for most cases as the concept of interest accumulating only at discrete intervals.

Equations (7-6) and (7-9) form the basis for developing continuous interest relationships. The symbol r represents the nominal interest rate with m interest periods per year. If the interest is compounded continuously, m approaches infinity. For N years Eq. (7-6) can be written as

$$F = P \lim_{m\to\infty} \left(1 + \frac{r}{m}\right)^{mN} = P \lim_{m\to\infty} \left(1 + \frac{r}{m}\right)^{(m/r)(rN)} \tag{7-10}$$

The fundamental definition for the base of the natural system of logarithms ($e = 2.71828$) is[†]

$$\lim_{m\to\infty} \left(1 + \frac{r}{m}\right)^{m/r} = e = 2.71828\cdots \tag{7-11}$$

†See, for example, W. Fulks, *Advanced Calculus*, 3d ed., J. Wiley, New York, 1978, pp. 55–56.

Thus[†]

$$F = Pe^{rN} \tag{7-12}$$

The term e^{rN} is known as the *continuous single-payment compound amount* factor, or the *continuous single-payment future-worth* factor. In functional form[‡]

$$e^{rN} = (F/P, r, N) \tag{7-13}$$

thus,

$$F = P(F/P, r, N) \tag{7-14}$$

and the factor $(F/P, r, N)$ is the ratio of future worth F to present worth P, when compounding is continuous at a nominal rate of r per year for N years.

Based on Eq. (7-9), the effective interest rate can be expressed in terms of the continuous nominal interest rate as

$$i_{\text{eff}} = e^r - 1 \tag{7-15}$$

from which

$$r = \ln(1 + i_{\text{eff}}) \tag{7-16}$$

and therefore,

$$e^{rN} = (1 + i_{\text{eff}})^N \tag{7-17}$$

and

$$F = Pe^{rN} = P(1 + i_{\text{eff}})^N \tag{7-18}$$

EXAMPLE 7-2 Calculations with Continuous Interest Compounding

For the case of a nominal annual interest rate of 20 percent per year, determine

a. The total amount to which \$1 of initial principal would accumulate after 1 year with annual compounding and the effective annual interest rate.

b. The total amount to which \$1 of initial principal would accumulate after 1 year with monthly compounding and the effective annual interest rate.

c. The total amount to which \$1 of initial principal would accumulate after 1 year with daily compounding and the effective annual interest rate.

[†]The same result can be obtained by noting that, for the case of continuous compounding, the differential change of F with time equals the nominal, fractional continuous interest rate r times F, or $dF/dN = rF$. This expression may be separated and integrated as follows to give Eq. (7-12):

$$\int_P^F (dF/F) = r\int_0^N dN$$

$$\ln\left(\frac{F}{P}\right) = rN \qquad \text{or} \qquad F = Pe^{rN}$$

[‡]ANSI Z94.7-2000.

d. The total amount to which \$1 of initial principal would accumulate after 1 year with continuous compounding and the effective annual interest rate.

■ Solution

a. Using Eq. (7-3), $P = \$1.0$, $r = 0.20$, $N = 1$, and

$$r = i = i_{\text{eff}} = 0.20 \text{ or } 20\% \text{ because compounding is annual}$$

$$F = P(1 + i_{\text{eff}})^N = (1 + 0.20)^1 = \$1.20$$

b. With Eq. (7-6), $P = \$1.0$, $r = 0.20$, $m = 12$, and

$$F = P\left(1 + \frac{r}{m}\right)^m = (1.0)\left(1 + \frac{0.20}{12}\right)^{12} = \$1.2194$$

Using Eq. (7-9),

$$i_{\text{eff}} = \left(1 + \frac{r}{m}\right)^m - 1 = \left(1 + \frac{0.20}{12}\right)^{12} - 1 = 0.2194, \text{ or } 21.94\%$$

c. With Eq. (7-6), $P = \$1.0$, $r = 0.20$, $m = 365$, and

$$F = P\left(1 + \frac{r}{m}\right)^m = (1.0)\left(1 + \frac{0.20}{365}\right)^{365} = \$1.2213$$

Using Eq. (7-9),

$$i_{\text{eff}} = \left(1 + \frac{r}{m}\right)^m - 1 = \left(1 + \frac{0.20}{365}\right)^{365} - 1 = 0.2213, \text{ or } 22.13\%$$

d. With Eq. (7-12), $P = (\$1.0)$, $r = 0.20$, $n = 1$, and

$$F = Pe^{rN} = (1.0)e^{(0.20)(1)} = \$1.2214$$

With Eq. (7-15),

$$i_{\text{eff}} = e^r - 1 = e^{0.20} - 1 = 0.2214, \text{ or } 22.14\%$$

■

Example 7-2 illustrates that continuously compounded interest gives a result which agrees much better with the results from monthly and daily compounding than does the result given by annual compounding when the same interest rate is used for each case. On the other hand, if the effective and nominal rates agree according to Eq. (7-9) or (7-15), then each case will give the same annual result. These results emphasize the importance of determining the appropriate interest rate to be used in economic calculations.

COST OF CAPITAL

There are several possible sources of capital for business ventures, including loans, bonds, stocks, and corporate funds. Corporate funds, primarily from undistributed profits and depreciation accumulations, are usually a major source of capital for established businesses. Borrowed funds are often used to supply all or part of corporate investments. The interest paid on the portion of an investment that comes from loans is one of the costs of making a product. Interest paid on bonds is also a cost of doing business. The question sometimes arises as to whether interest on investor-owned funds can be charged as a cost of doing business. The answer, based on court decisions and income tax regulations, is definitely no. Therefore, interest on corporate funds and earnings paid on preferred or common stocks are not a manufacturing cost, because they are a return to owners of equity in the corporation. Furthermore, the borrowed principal, which is first a gain, but must be repaid, is not taxable as a gain; nor is repaying it deductible as a business expense.

In the preliminary design of a project, unless more specific information is available, one of the following two methods is usually employed to account for interest costs:

1. No interest costs are included. This assumes that all the necessary capital comes from internal corporate funds, and comparisons to alternative investments must be on the same basis.
2. Interest is charged on the total capital investment, or a predetermined fraction thereof, at a set interest rate, usually equivalent to rates charged for bank loans.

As the design proceeds to the final stages, the actual sources of new capital should be considered in detail, and more appropriate interest costs can then be used.

Because of the different methods used for treating interest costs, a definite statement should be made concerning the particular method employed in any given economic analysis. Interest costs become especially important when comparisons are made among alternative investments. These comparisons as well as the overall cost picture are simplified if the role of interest in the analysis is clearly defined.

Income Tax Effects

The effect of income taxes on the cost of capital is very important. In determining income taxes, interest on loans and bonds can be considered as a cost, while the return of both preferred and common stock cannot be included as a cost. If the incremental annual income tax rate is 35 percent, every \$1 spent for interest on loans or bonds has a true cost after taxes of only \$0.65. Thus, after income taxes are taken into consideration, a bond issued at an annual interest rate of 5 percent actually has an interest rate of only $(5)(65/100) = 3.25$ percent. On the other hand, dividends on preferred stock must be paid from net profits after taxes. If a preferred stock has an annual dividend rate of 8 percent, the equivalent rate before income taxes is $(8)(100/65) = 12.3$ percent, and after income taxes it is $(12.3)(65/100) = 8$ percent.

A comparison of representative interest and dividend rates for different types of externally financed capital is presented in Table 7-1.

The choice of capital sources to be used to fund a project is usually made at the highest corporate level based upon corporate circumstances and policies. It would be typical for a new business to rely primarily on external financing, while a successful, established business would use a higher proportion of internal funds.

Table 7-1 Representative costs for externally financed capital†

Source of capital	Indicated interest or dividend rate, %/yr	Interest or dividend rate before taxes, %/yr	Interest or dividend rate after taxes, %
Bonds	5	5	3.25
Bank or other loans	8	8	5.2
Preferred stock	8	12.3	8
Common stock	n/a	13.8	9

†Income tax rate of 35 percent of taxable income.

Loan Payments

There are many types of loan repayment terms. A very common type, used for nearly all home mortgages and many business loans, calls for constant periodic payments for a fixed period. Each payment covers the current interest due and repays some of the remaining principal balance. The total payment is constant, but the principal balance decreases, so that the interest portion of each payment is smaller than the previous one and the principal portion of each payment is larger than the previous one. Because this loan type is so common, the equations used to calculate the payment will be developed and illustrated.

The constant amount that must be paid each period so that the required interest is paid and the original amount borrowed is returned over the total life of the loan is given by

$$L = I_j + p_j \tag{7-19}$$

where L is the constant payment each period, I_j the jth period interest payment, and p_j the jth period principal payment. The index j begins at 1, because payment is made at the end of the period. [The symbol i in the equations in this section represents the effective annual interest rate for annual payments or the nominal rate per period (r/m) for other periods.] The interest payment is

$$I_j = iP_{j-1} \tag{7-20}$$

where i is the interest rate and P_{j-1} the principal balance after payment $j-1$. The remaining principal balance after $j-1$ periods is

$$P_{j-1} = P_0 - \sum_{m=1}^{j-1} p_m \tag{7-21}$$

where p_m is the mth principal payment and P_0 the initial amount of the loan. From Eqs. (7-19), (7-20), and (7-21),

$$p_j = L - I_j = L - i\left(P_0 - \sum_{m=1}^{j-1} p_m\right) \tag{7-22}$$

From Eq. (7-22),

$$p_1 = L - iP_0 \tag{7-23}$$

similarly,

$$p_2 = L - i(P_0 - p_1) = L - i[P_0 - (L - iP_0)] = L(1+i) - iP_0(1+i) \tag{7-24}$$

and

$$\begin{aligned} p_3 &= L - i(P_0 - p_i - p_2) = L - i\{P_0 - (L - iP_0) - [L(1+i) - iP_0(1+i)]\} \\ &= L[(1+i) + i(1+i)] - iP_0[1 + i + i(1+i)] \end{aligned} \tag{7-25}$$

from which

$$p_3 = L(1+i)^2 - iP_0(1+i)^2 \tag{7-26}$$

In general,

$$p_j = L(1+i)^{j-1} - iP_0(1+i)^{j-1} \tag{7-27}$$

The sum of all the principal payments must equal the original loan principal (i.e., pay off the original loan). Accordingly,

$$\begin{aligned} \sum_{j=1}^{N} p_j = P_0 &= \sum_{1}^{N} L(1+i)^{j-1} - \sum_{1}^{N} iP_0(1+i)^{j-1} \\ &= L\sum_{1}^{N}(1+i)^{j-1} - iP_0\sum_{1}^{N}(1+i)^{j-1} \end{aligned} \tag{7-28}$$

Solving for L gives

$$L = \frac{P_0\left[1 + i\sum_{1}^{N}(1+i)^{j-1}\right]}{\sum_{1}^{N}(1+i)^{j-1}} \tag{7-29}$$

After first calculating the summation term in Eq. (7-29), we can use this equation to calculate the constant payment amount for any loan amount, interest rate, and total payment period. Equations (7-20) and (7-22) can then be used, in that order, to calculate the interest and principal portions of each payment. Equation (7-21) can be used to calculate the remaining principal after each payment. These calculations are repeated for each payment until the end of the period. Example 7-3 illustrates these calculations.

EXAMPLE 7-3 Calculation of Loan Payments with Annually Compounded Interest

Present a spreadsheet for the following calculations: A loan of \$100,000 at a nominal interest rate of 10 percent per year is made for a repayment period of 10 years. Determine the constant payment per period, the interest and principal paid each period, and the remaining unpaid principal at the end of each period by using constant end-of-month payments (assume 12 equal-length months per year).[†] Repeat for annual, end-of-the-year payments.

■ Solution

In both cases the procedure is as follows:

Calculate the constant payment L from Eq. (7-29), after first calculating the summation of the $(1 + i)^{j-1}$ term. Then calculate the interest payment for the first period, using Eq. (7-20). Determine the principal payment for the first period by subtracting the interest payment from the total constant payment, using Eq. (7-22). Establish the remaining unpaid balance by subtracting the first principal payment from the original principal amount, Eq. (7-21). Repeat this procedure period by period until the full term of the loan is reached. The results are provided in the following tables.

Check: If the calculations have been done correctly, the final remaining principal will equal zero.

Monthly loan payments

P_0 = \$100,000.00; *nominal interest* $r = 0.1$ *fraction/yr; payments/yr* = 12, *for 10 yr*

Month j	$(1 + 0.1/12)^{j-1}$ Column sum = 204.84498	Constant payment L, \$/month, Eq. (7-29)	Interest payment, \$/month, Eq. (7-20)	Principal payment, \$/month, Eq. (7-22)	Remaining principal, \$, Eq. (7-21)
0					100,000.00
1	1.0000000	1,321.51	833.33	488.17	99,511.83
2	1.0083333	1,321.51	829.27	492.24	99,019.58
3	1.0167361	1,321.51	825.16	496.34	98,523.24
4	1.0252089	1,321.51	821.03	500.48	98,022.76
5	1.0337523	1,321.51	816.86	504.65	97,518.11
6	1.0423669	1,321.51	812.65	508.86	97,009.25
7	1.0510533	1,321.51	808.41	513.10	96,496.15
8	1.0598121	1,321.51	804.13	517.37	95,978.78
9	1.0686439	1,321.51	799.82	521.68	95,457.10
10	1.0775492	1,321.51	795.48	526.03	94,931.07
11	1.0865288	1,321.51	791.09	530.42	94,400.65
12	1.0955832	1,321.51	786.67	534.84	93,865.82
From here, most results are deleted; only those for every 12th month are shown.					
24	1.2103051	1,321.51	730.67	590.84	87,089.30
36	1.3370398	1,321.51	668.80	652.71	79,603.20
48	1.4770454	1,321.51	600.45	721.06	71,333.20
60	1.6317113	1,321.51	524.95	796.56	62,197.23
72	1.8025728	1,321.51	441.54	879.97	52,104.60
84	1.9913258	1,321.51	349.39	972.11	40,955.15
96	2.1998436	1,321.51	247.60	1,073.91	28,638.19
108	2.4301960	1,321.51	135.15	1,186.36	15,031.50
120	2.6846692	1,321.51	10.92	1,310.59	0.00
	Total payments	\$158,580.88	Total interest paid \$58,580.88	Total principal paid \$100,000.00	

[†]*Note:* At a 50 percent per year interest compounded monthly, the error caused by this assumption is less than 8×10^{-4} percent in 1 year and 8×10^{-3} percent in 10 years. The error increases with increasing interest rate and length of period, but it takes extraordinarily large values of interest or time to make this error significant.

The only changes from the monthly payment are that i_{eff} is 0.10 and N is 10.

P_0 = \$100,000.00; *interest i* = 0.1 *fraction/yr; payments/yr* = 1, *for 10 yr*

Year j	$(1 + 0.1)^{j-1}$ Column sum = 15.937425	Constant payment, \$/yr, Eq. (7-29)	Interest payment, \$/yr, Eq. (7-20)	Principal payment, \$/yr, Eq. (7-22)	Remaining principal, \$, Eq. (7-21)
0					100,000.00
1	1.000000	16,274.54	10,000.00	6,274.54	93,725.46
2	1.100000	16,274.54	9,372.55	6,901.99	86,823.47
3	1.210000	16,274.54	8,682.35	7,592.19	79,231.27
4	1.331000	16,274.54	7,923.13	8,351.41	70,879.86
5	1.464100	16,274.54	7,087.99	9,186.55	61,693.31
6	1.610510	16,274.54	6,169.33	10,105.21	51,588.10
7	1.771561	16,274.54	5,158.81	11,115.73	40,472.37
8	1.948717	16,274.54	4,047.24	12,227.30	28,245.07
9	2.143589	16,274.54	2,824.51	13,450.03	14,795.04
10	2.357948	16,274.54	1,479.50	14,795.04	0.00
		Total payments \$162,745.39	Total interest paid \$62,745.39	Total principal paid \$100,000.00	

In this example, 7.1 percent less total interest is required by paying monthly compared to annually. In general, the more frequently loan payments are made, the less total interest is paid.

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TIME VALUE OF MONEY

Money can be used to earn money by investment, for example, in savings accounts, bonds, stocks, or projects. Therefore, an initial amount of money that is invested increases in value with time. This effect is known as the *time value of money*. By virtue of this earning capacity, an amount of money available now is worth, or equivalent to, a greater amount in the future. For example, \$1000 invested at 10 percent per year compounded annually is worth \$1100 one year later and \$2594 ten years later. The value at a future time is the *future worth* (also known as the *future value*) of the money. This earning power of money can be included in the analysis of business profitability. Methods for calculating the worth of money at different times are similar to those for calculating interest.

An amount of money available at some time in the future is worth, or equivalent to, a smaller amount at present. For example, \$1100 available at the end of 1 year after being compounded at 10 percent over that 1-year period is actually only worth \$1000 at present. The \$1000 is the *present worth* (also known as *present value*) of the future \$1100. This comparison again exemplifies the concept of the time value of money. These amounts—\$1000 now, \$1100 one year from now, and \$2594 ten years from now—are all said to be *equivalent* at a 10 percent rate of interest, and only at that rate, compounded annually. Amounts of money are equivalent if they are equal when calculated at the same time with a specified interest rate.

The time value of money often is a very important consideration when investments are compared that require or generate different amounts of funds at different times. In fact, the timing of expenses and income may significantly impact the present

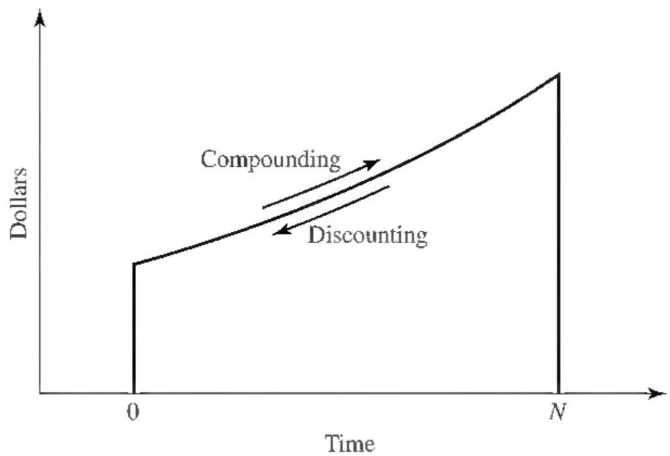

Figure 7-2
Schematic of compounding and discounting

worth of such funds. Put another way, the most appropriate way to make economic comparisons is to make all cash flows equivalent. Thus, it is necessary to be able to determine the value of investments at any selected time.

The time value of money is related solely to the capacity of money to earn money. It has nothing to do with inflation. Inflation, or deflation, refers to the change in the prices of goods and services, not to the amount of money available. For example, if the rate of inflation is 4 percent per year and if the cost of an item is $100 now, it will cost $104 one year from now—regardless of the time value of money.[†]

The calculation of the future worth of a present amount of money, known as compounding, was illustrated earlier in this chapter. The inverse of compounding that is, obtaining the present worth of a future amount, is designated as *discounting*. The expression for discrete compounding of a single present amount to obtain its future worth was presented in Eq. (7-3) or (7-5). The equation for discrete discounting of a single future amount to obtain its present worth is obtained by rearranging Eq. (7-3) to give

$$P = F(1+i)^{-N} \tag{7-30}$$

This can also be expressed in terms of the ANSI functional form of the discount factor as

$$P = F(P/F, i, N) \tag{7-30a}$$

hence the *discrete single-payment discount* factor or the *discrete single-payment present-worth* factor is

$$(P/F, i, N) = (1+i)^{-N} \tag{7-31}$$

The operations of compounding and discounting are inverses of each other. The present-worth factor of Eq. (7-31) is the inverse of the future-worth factor of Eq. (7-4). Figure 7-2 illustrates that compounding and discounting are inverse operations.

The future worth of a present amount of money, with continuous compounding, was given by Eq. (7-12) or (7-14). The equation for discounting future values to the present, obtained by rearranging Eq. (7-12), is therefore

$$P = Fe^{-rN} \tag{7-32}$$

[†]The effect of inflation on economic analyses is discussed in Chap. 8.

or, by equating Eqs. (7-12) and (7-14) and solving,

$$(P/F, r, N) = e^{-rN} \tag{7-33}$$

In the ANSI form, the distinction between discrete and continuous compounding or discounting is the replacement of i in the discrete compounding form with r for the continuous compounding.

Comparison of the equations involved emphasizes that discounting is the inverse of compounding and that the discount factors $(P/F, i, N)$ and $(P/F, r, N)$ are the inverse of the compounding factors $(F/P, i, N)$ and $(F/P, r, N)$, respectively. Discounting is no more difficult than compounding. It is merely the calculation of a present amount from a future amount based upon the capacity of money to earn money.

CASH FLOW PATTERNS

Cash flow was introduced in Chap. 6 and is the amount of funds that enter the corporate treasury as a result of the activities of the project. The annual cash flow is equal to the net (after-tax) profit plus the allowed depreciation charges for the year. Since cash flows of a project occur over the lifetime of a project, it is necessary to convert them to equivalent values. This is done either by discounting future cash flows or by compounding earlier cash flows to a particular point in time. While it is essential that all cash flows be converted to the same time, the time selected is not critical, because different present-worth amounts at any one time will be in the same rank order and in the same ratio as at any other time. Often the selected time is not the present, but rather sometime in the future. It is common, while in the design phase of a project, to select the projected start-up time as the time at which all "present" values are calculated.

Discrete Cash Flows

Cash flow patterns are often best perceived graphically. Figure 7-3 shows equal discrete cash flows occurring once per month at the end of the month for a period of 1 year; each cash flow is represented by a bar.

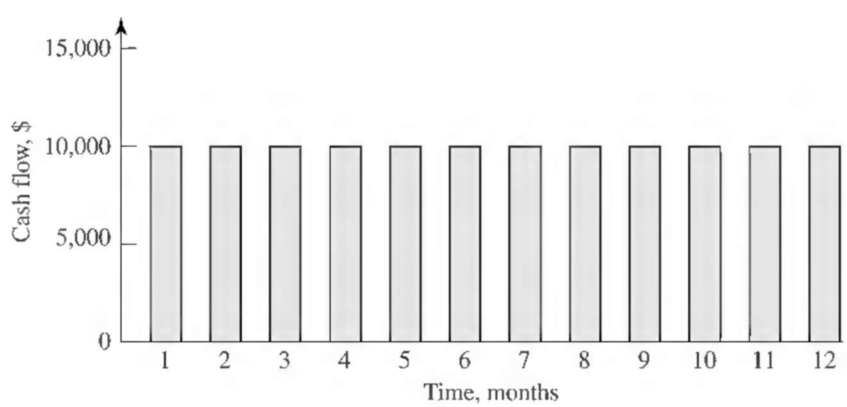

Figure 7-3
Constant, end-of-month cash flows for 1 yr

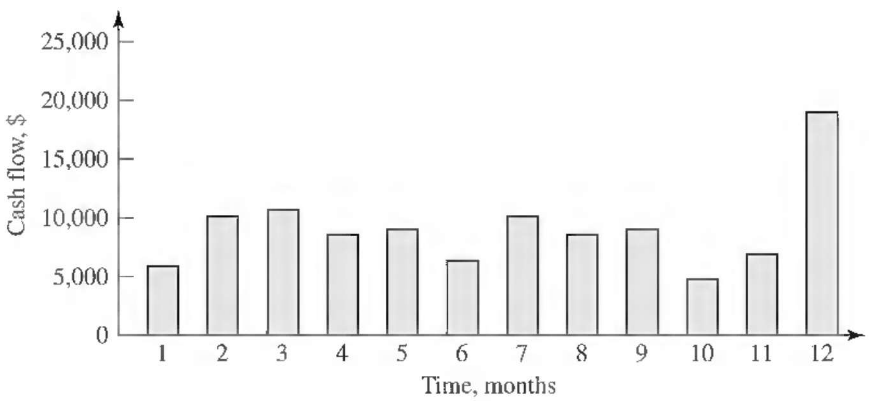

Figure 7-4
Series of unequal end-of-month cash flows

Figure 7-4 shows unequal cash flows occurring at the end of each month for 1 year. The cash flows shown in Figs. 7-3 and 7-4 are equivalent at a discount rate of 10 percent per year. That is, they have the same worth at a particular time when calculated at that discount rate, whether that worth is calculated at time zero, at 12 months, or at any other time. It follows that if two different cash flows are equivalent at one interest rate, they are not equivalent at any other interest rate.

Present Worth of Two Different Cash Flow Patterns — EXAMPLE 7-4

The cash flow patterns in Figs. 7-3 and 7-4 are shown numerically in the second column of the two spreadsheets shown below. With interest compounded monthly at a rate of 10 percent per year, calculate the total amount of the cash flow, the present worth at zero time, and the future worth at 12 months for both series of cash flows.

■ Solution

Equal monthly cash flows

End of month j	Cash flow, $	Present-worth factor = $(1 + 0.1/12)^{-j}$	Present worth, $, @ 0 mo	Future-worth factor = $(1 + 0.1/12)^{(12-j)}$	Future worth, $, @ 12 mo
1	10,000	0.9917	9,917	1.0956	10,956
2	10,000	0.9835	9,835	1.0865	10,865
3	10,000	0.9754	9,754	1.0775	10,775
4	10,000	0.9673	9,673	1.0686	10,686
5	10,000	0.9594	9,594	1.0598	10,598
6	10,000	0.9514	9,514	1.0511	10,511
7	10,000	0.9436	9,436	1.0424	10,424
8	10,000	0.9358	9,358	1.0338	10,338
9	10,000	0.9280	9,280	1.0252	10,252
10	10,000	0.9204	9,204	1.0167	10,167
11	10,000	0.9128	9,128	1.0083	10,083
12	10,000	0.9052	9,052	1.0000	10,000
Total	$120,000	Total	$113,745	Total	$125,656

Unequal monthly cash flows

End of month j	Cash flw , $	Present-worth factor = $(1 + 0.1/12)^{-j}$	Present worth, $, @ 0 mo	Future-worth factor = $(1 + 0.1/12)^{(12-j)}$	Future worth, $, @ 12 mo
1	7,000	0.9917	6,942	1.0956	7,669
2	11,000	0.9835	10,819	1.0865	11,952
3	11,500	0.9754	11,217	1.0775	12,392
4	9,500	0.9673	9,190	1.0686	10,152
5	10,000	0.9594	9,594	1.0598	10,598
6	7,500	0.9514	7,136	1.0511	7,883
7	11,000	0.9436	10,379	1.0424	11,466
8	9,500	0.9358	8,890	1.0338	9,821
9	10,000	0.9280	9,280	1.0252	10,252
10	6,000	0.9204	5,522	1.0167	6,100
11	8,000	0.9128	7,302	1.0083	8,067
12	19,304	0.9052	17,474	1.0000	19,304
Total	$120,304	Total	$113,745	Total	$125,656

The results show that the present and future worths are the same for both cash flows; therefore, the two cash flows are equivalent at the specified interest rate. This is true even though the patterns are very different and the actual totals of the two cash flows are somewhat different.

Annual compounding and discounting are often used in project evaluation. It is also common to represent total annual cash flows as a single (discrete) amount received at the end of the year (year-end convention). For periods other than 1 year, 1 month for example, the end of the period is usually used as the time for the discrete cash flow. In present-worth calculations, these are approximations for cash flows that occur frequently over the year.

Continuous Cash Flows

A continuous cash flow is one in which receipts and expenditures occur continuously over time. This is an idealized approximation of cash flows that occur frequently compared to once per year. Good business practice dictates investments of available funds as soon as they are received. Therefore, in the case of a continuous cash flow, the cash flow is invested continuously as it is received. If interest is compounded continuously, the rate of earning at any instant consists of two terms: (1) a continuous, constant rate of cash flow $\bar{P}$ in dollars per period (usually 1 year) and (2) the compound rate of earning on the accumulated amount M that has been invested at the rate r. This can be expressed as

$$\frac{dM}{d\theta} = \bar{P} + rM \tag{7-34}$$

Rearranging and forming the integrals give

$$\int_0^F \frac{dM}{\bar{P} + rM} = \int_{j-1}^{j} d\theta \tag{7-35}$$

where F is the future worth of the cash flow plus the earnings at the end of a 1-year period. Performing the integration gives

$$\frac{1}{r} \ln \frac{\bar{P} + rF}{\bar{P}} = j - (j - 1) = 1 \qquad (7\text{-}36)$$

In exponential form this can be simplified to

$$\frac{\bar{P} + rF}{\bar{P}} = e^r \qquad (7\text{-}37)$$

and solving for F results in

$$F = \bar{P}\left(\frac{e^r - 1}{r}\right) \qquad (7\text{-}38)$$

The term $(e^r - 1)/r$ multiplied by the cash flow rate $\bar{P}$ equals the amount of funds accumulated at the end of the year from a 1-year, continuous, constant cash flow invested at a rate r. It can be represented as a compounding factor in ANSI functional form as

$$\frac{e^r - 1}{r} = (F/\bar{P}, r, j) \qquad (7\text{-}39a)$$

The end-of-year N future-worth factor for such a cash flow is

$$(F/\bar{P}, r, N) = \left(\frac{e^r - 1}{r}\right) e^{(N-j)} \qquad (7\text{-}39b)$$

The present worth of a 1-year, continuous, constant cash flow starting at the end of year $j - 1$ and ending at the end of year j, with continuous discounting, is determined by multiplying Eq. (7-38) by the discount factor e^{-rj} of Eq. (7-32) to give

$$P = \bar{P}\left(\frac{e^r - 1}{r}\right) e^{-rj} \qquad (7\text{-}40)$$

The present-worth factor for this case is

$$(P/\bar{P}, r, j) = \left(\frac{e^r - 1}{r}\right) e^{-rj} \qquad (7\text{-}41)$$

The term $(e^r - 1)/r$ in Eq. (7-41) accounts for earnings from the cash flow that was invested as it was received over the year. It is this term which distinguishes the continuous interest, continuous cash flow discounting factor from the continuous interest, discrete, year-end cash flow factor given by Eq. (7-33). The term e^{-rj} in Eq. (7-41) discounts the worth of the accumulated funds at the end of the year to time zero, just as it does in Eq. (7-33). It is possible to use different interest rates for the r in the two components of the discount factor in Eq. (7-41). However, the same interest rate is generally used in both terms.

For a cash flow occurring over a period of N years, starting at time zero, the future-worth factor is

$$(F/\bar{A}, r, N) = \frac{e^{rN} - 1}{r} \tag{7-42}$$

where $\bar{A}$ is a continuous, equal cash flow occurring in each year (or period) over N years (or periods). The present-worth factor of such a cash flow is

$$(P/\bar{A}, r, N) = \left(\frac{e^{rN} - 1}{r}\right) e^{-rN} \tag{7-43}$$

EXAMPLE 7-5 Continuous Cash Flow Calculations

Find the future worth at the end of the cash flow period and the present worth at the beginning of the cash flow period of a constant, continuous cash flow of $450,000 per year for 6 years. Use a continuously compounded interest rate of 8 percent per year. Determine the future worth and present worth, using the simplest approach. Also tabulate the cash flow a year at a time, using Eqs. (7-39*b*) and (7-41).

■ Solution

Calculate the future worth at 6 years with Eq. (7-42):

$$F = \bar{A}(F/\bar{P}, r, N) = \bar{A}\left(\frac{e^{rN} - 1}{r}\right)$$

$$= \$450{,}000 \frac{e^{(0.08)(6)} - 1}{0.08} = \$3.465 \times 10^6$$

Notice that the total of the cash flow in 6 years of $450,000 annually is $2.7 million. The additional $0.765 million shown in the future-worth calculation is interest earned on the cash flow itself and by compounding the earned interest.

The present worth at time zero of the cash flow is given by Eq. (7-43):

$$P = \bar{A}\left(\frac{e^{rN} - 1}{r}\right) e^{-rN}$$

Notice that the same result can be obtained by multiplying the future worth obtained with Eq. (7-42) by the discount factor e^{-rN}; thus

$$P = Fe^{-rN}$$

$$= (\$3.465 \times 10^6)(e^{-(0.08)(6)}) = \$2.144 \times 10^6$$

Tabulation of the cash flow a year at a time with a continuous cash flow of $450,000 per year is presented here.

Year	$\bar{P}$, $1000/yr	$F/\bar{P}$, yr, Eq. (7-39*b*)	Future worth *F* at 6 yr, $1000	*P*/*F*, yr at 6 yr, Eq. (7-33)	Present worth *P* at 0 yr, $1000
1	450	1.5531	699	0.6188	432
2	450	1.4337	645	0.6188	399
3	450	1.3235	596	0.6188	369
4	450	1.2217	550	0.6188	340
5	450	1.1278	508	0.6188	314
6	450	1.0411	468	0.6188	290
Totals	$2700		$3465		$2144

Notice that these values agree with those obtained directly by using Eqs. (7-42) and (7-43). The use of these equations is much simpler for this example. However, the year-by-year method is useful for the more common situation, where the cash flow changes from year to year. The present worth also can be found directly one year at a time using Eq. (7-40).

COMPOUNDING AND DISCOUNTING FACTORS

Several compounding and discounting factors have already been introduced. Cash flows and interest compounding can be either discrete or continuous. This gives four possible combinations of cash flow and interest types, as shown in Table 7-2.

The period of discrete compounding can be any time interval, and a discrete cash flow can occur at any time throughout the period. The most common choices are a compounding period of 1 year and the year-end convention for cash flow. These two choices will be used for discrete times and cash flows throughout the rest of this chapter, unless otherwise indicated.

For simple cash flow patterns, such as a single amount as shown in Fig. 7-5 or equal flows for a period of years as presented in Fig. 7-6, it is possible to derive compounding and discount factors. Some of these were derived earlier in this chapter. The rest of these factors will be presented without derivation and are available from many

Table 7-2 Combinations of cash flow and interest types

	Interest compounding	
Cash flow pattern	Annual	Continuous
Annual, end of year	Case 1	Case 2
Continuous	Case 3	Case 4

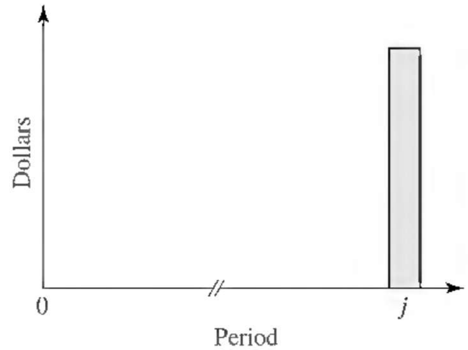

Figure 7-5
Single-amount cash flow

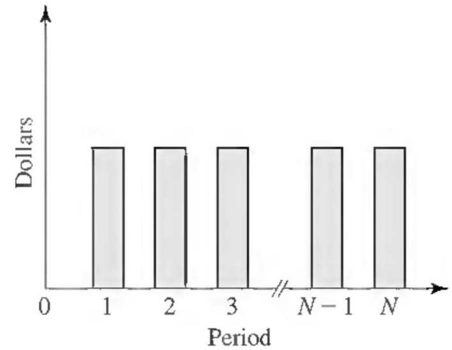

Figure 7-6
Uniform series cash flow

Table 7-3 Compounding and discounting factors for discrete interest compounding and discrete or continuous cash flows

	Compounding			Discounting		
Cash flw pattern	**Factor name; description; units**	**Symbol**	**Formula**	**Factor name; description; units**	**Symbol**	**Formula**
Single amount at end of year *j*. See Fig. 7-5.	Future worth; multiplies a single amount at time *j* to give a single amount at *N*; dimensionless	$(F/P, i, N-j)$	$(1+i)^{N-j}$	Present worth; multiplies a single amount at time *j* to give a single amount at time zero; dimensionless	$(P/F, i, j)$	$(1+i)^{-j}$
Series of equal amounts from time 1 to *N*; uniform series. See Fig. 7-6.	Future worth; multiplies the annual rate of a series of *N* equal amounts to give a single amount at *N*; years	$(F/A, i, N)$	$\frac{(1+i)^N-1}{i}$	Sinking fund; multiplies a single amount at *N* to give the annual rate of a series of *N* equal amounts; $(\text{years})^{-1}$	$(A/F, i, N)$	$\frac{i}{(1+i)^N-1}$
Series of equal amounts from time 1 to *N*; uniform series. See Fig. 7-6.	Capital recovery; multiplies a single amount at time zero to give the annual rate of a series of *N* equal amounts; $(\text{years})^{-1}$	$(A/P, i, N)$	$\frac{i(1+i)^N}{(1+i)^N-1}$	Present worth; multiplies the annual rate of a series of *N* equal amounts to give a single amount at time zero; years	$(P/A, i, N)$	$\frac{(1+i)^N-1}{i(1+i)^N}$

sources.† Table 7-3 shows the common compounding and discounting factors for case 1, discrete interest and discrete cash flow. These same factors are also applicable to case 3 since the final result is independent of the pattern of cash flow over the year when interest for a year is calculated based on the amount of principal at the beginning of the year.

Present Worth of Annual Cash Flows with Annual Compounding — EXAMPLE 7-6

A cash flow consisting of \$10,000 per year is received in one discrete amount at the end of each year for 10 years. Interest will be at 10 percent per year compounded annually. Determine the present worth at time zero and the future worth at the end of 10 years of this cash flow.

■ Solution

Identify the cash flow pattern as a series of discrete, equal amounts received at the end of years 1 to N, with annual interest compounding. In Table 7-3, the factor to use to obtain the present worth of this series is $(P/A, i, N)$. Notice that the factor has units of years.

With $A = \$10{,}000/\text{yr}$, $i = 0.1$, and $N = 10$, the present worth is

$$P = A(P/A, i, N) = \frac{A[(1+i)^N - 1]}{i(1+i)^N} = \frac{10{,}000[(1.1)^{10} - 1]}{0.1(1.1)^{10}}$$

$$= 10{,}000(6.144567) = \$61{,}445.67, \text{ or } \$61{,}446$$

The total cash flow over the 10 years is 10(\$10,000), or \$100,000. This present worth reflects the fact that \$61,446 invested at time zero would grow to the same amount—the future worth calculated below—in 10 years, as does the actual cash flow.

From Table 7-3 the factor used to get the future worth from the annual amount is $(F/A, i, N)$. Alternatively, the future worth may be obtained by multiplying the present worth P by the factor $(F/P, i, N)$. The same result is obtained either way,

$$F = P(F/P, i, N) = P(1+i)^N = \$61{,}446(1.1)^{10} = \$159{,}375$$

Notice that the \$100,000 received and invested over the 10 years is equivalent to \$61,446 invested at time zero, and is also equivalent to \$159,375 received 10 years later.

The results would be the same even if the cash flow were continuous over each year, provided that interest was compounded on the principal at the beginning of the year (case 3).

■

Compounding and discounting factors for case 2, continuous interest, and annual cash flows are given in Table 7-4.

†For example, see ANSI Z94.7-2000; J. A. White, M. H. Agee, and K. E. Case, *Principles of Engineering Economic Analysis,* 2d ed., J. Wiley, New York, 1977; and M. S. Peters and K. D. Timmerhaus, *Plant Design and Economics for Chemical Engineers,* 4th ed., McGraw-Hill, New York, 1991.

Table 7-4 Compounding and discounting factors for continuous interest compounding and discrete cash flows

	Compounding			Discounting		
Cash flow pattern	Factor name; description; units	Symbol	Formula	Factor name; description; units	Symbol	Formula
Single amount at end of year *j*. See Fig. 7-5.	Future worth; multiplies a single amount at *j* to give a single amount at *N*; dimensionless	$(F/P, r, N-j)$	$e^{r(N-j)}$	Present worth; multiplies a single amount at *j* to give a single amount at zero; dimensionless	$(P/F, r, j)$	e^{-rj}
Series of equal amounts from time 1 to *N*; uniform series. See Fig. 7-6.	Future worth, annuity; multiplies the annual rate of a series of *N* equal amounts to give a single amount at *N*; years	$(F/A, r, N)$	$\frac{e^{rN}-1}{e^r-1}$	Sinking fund; multiplies a single amount at *N* to give the annual rate of a series of equal amounts; $(\text{years})^{-1}$	$(A/F, r, j)$	$\frac{e^r-1}{e^{rN}-1}$
Series of equal amounts from time 1 to *N*; uniform series. See Fig. 7-6.	Capital recovery; multiplies a single amount at time zero to give the annual rate of a series of *N* equal amounts; $(\text{years})^{-1}$	$(A/P, r, N)$	$\frac{e^{rN}(e^r-1)}{e^{rN}-1}$	Present worth; multiplies the annual rate of a series of equal amounts to give a single amount at time zero; years	$(P/A, r, j)$	$\frac{e^{rN}-1}{e^{rN}(e^r-1)}$

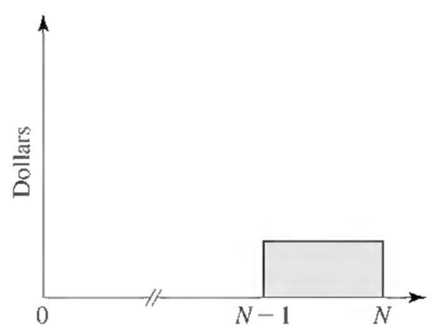

Figure 7-7
One-year continuous constant-rate cash flow

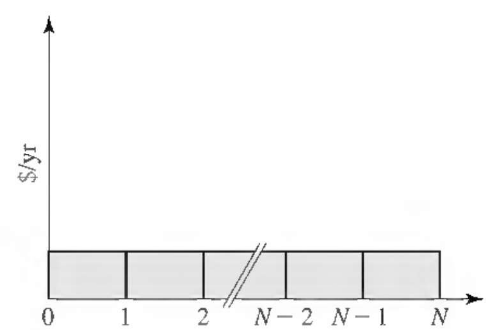

Figure 7-8
Series of continuous, constant-rate cash flows

Future and Present Worths of Discrete Annual Cash Flows Compounded Continuously

EXAMPLE 7-7

Repeat Example 7-6 but require that the compounding be performed continuously. Do the calculations two ways: first using the nominal interest rate $r = 0.1$ and then using $i_{\text{eff}} = 0.1$.

■ Solution

The cash flow pattern is the same as in Example 7-6, but the appropriate equations are from Table 7-4. The future worth is given by

$$F = \bar{A}\left(\frac{e^{rN} - 1}{e^{r} - 1}\right)$$

$$= \$10{,}000\left(\frac{e^{1.0} - 1}{e^{0.1} - 1}\right) = \$163{,}380$$

The present worth is found by using

$$P = \bar{A}\left(\frac{e^{rN} - 1}{e^{r} - 1}\right) e^{-rN}$$

$$= Fe^{-rN}$$

$$= \$163{,}380(e^{-1.0}) = \$60{,}104$$

Notice that these values are somewhat different from those calculated in Example 7-6. If $i_{\text{eff}} = 0.1$, then $r = \ln(1 + i_{\text{eff}})$

$$r = \ln(1 + 0.1) = 0.09531$$

which gives

$$F = \$10{,}000\left(\frac{e^{0.9531} - 1}{e^{0.09531} - 1}\right) = \$159{,}375$$

and

$$P = (\$159{,}375)(e^{-0.9531}) = \$61{,}446$$

Both of these values agree with those obtained in Example 7-6. ■

The compounding and discounting factor for case 4, continuous interest and continuous cash flows, are given in Table 7-5. Figure 7-7 illustrates a 1-year, continuous, constant rate cash flow and Fig. 7-8 shows a series of continuous, constant cash flow rates for N years.

Table 7-5 Compounding and discounting factors for continuous interest compounding and continuous cash flows

	Compounding			Discounting		
Cash flw pattern	**Factor name; description; units**	**Symbol**	**Formula**	**Factor name; description; units**	**Symbol**	**Formula**
Continuous, constant rate for 1 year only, ending at the end of year j. See Fig. 7-7.	Future worth; multiplies continuous annual rate to give a single amount at N; dimensionless	$(F/\bar{P}, r, N)$	$\frac{e^r - 1}{r} e^{r(N-j)}$	Present worth; multiplies continuous annual rate to give a single amount at time zero; dimensionless	$(P/\bar{P}, r, j)$	$\left(\frac{e^r - 1}{r}\right) e^{-rj}$
Series of continuous, constant annual amounts from time zero to N. See Fig. 7-8.	Future worth, multiplies the annual rate of a series of continuous, equal amounts to give a single amount at N; years	$(F/\bar{A}, r, N)$	$\frac{e^{rN} - 1}{r}$	Sinking fund; multiplies a single amount at N to give the annual rate of a series of continuous, equal amounts, $(\text{years})^{-1}$	$(\bar{A}/F, r, N)$	$\frac{r}{e^{rN} - 1}$
Series of continuous, constant annual amounts from time zero to N. See Fig. 7-8.	Capital recovery; multiplies a single amount at time zero to give the annual rate of a series of continuous, equal amounts; $(\text{years})^{-1}$	$(\bar{A}/P, r, N)$	$\frac{re^{rN}}{e^{rN} - 1}$	Present worth; multiplies the annual rate of a series of continuous, equal amounts to give a single amount at time zero; years	$(P/\bar{A}, r, N)$	$\frac{e^{rN} - 1}{r} e^{-rN}$

EXAMPLE 7-8 Worth of a Capital Investment Converted to a Continuous Constant Annual Series

Use a continuous nominal earning rate of 12 percent per year.

a. Find the future worth at start-up (time zero) of an investment made according to this schedule: \$3 million is invested continuously at a constant rate for 1 year, beginning 2 years before start-up; \$7 million is invested continuously at a constant rate for 1 year, beginning 1 year before start-up.

b. Convert the worth of the investment at start-up to a continuous, constant annual series for a 10-year period.

■ **Solution**

The investment and annual cash flow patterns can schematically be shown below in Fig. 7-9:

Figure 7-9

a. The two investments are converted to their worth at time zero by the future worth of a continuous cash flow factor from Table 7-5.

$$F = \bar{P}\left(\frac{e^r - 1}{r}\right)e^{r(N-j)}$$

So

$$F_1 = (3 \times 10^6)\left(\frac{e^{0.12} - 1}{0.12}\right)e^{0.12[0-(-2)]} = \$4.052 \times 10^6$$

and

$$F_2 = (7 \times 10^6)\left(\frac{e^{0.12} - 1}{0.12}\right)e^{0.12[0-(-1)]} = \$8.386 \times 10^6$$

The worth of the investment at start-up is the sum of these two values and is $\$12.44 \times 10^6$. Because the \$10 million invested could have been earning 12 percent per year, its worth at start-up is greater than the amount actually spent.

b. To convert the worth at zero time to an annual series, use the capital recovery factor from Table 7-5.

$$\bar{A} = P\left(\frac{re^{rN}}{e^{rN} - 1}\right)$$

$$\bar{A} = (\$12.44 \times 10^6)\left(\frac{0.12e^{(0.12)(10)}}{e^{(0.12)(10)} - 1}\right) = \$2.14 \times 10^6 \text{ per year}$$

Notice that \$21.4 million must be earned over 10 years to be equivalent to the original \$10 million invested (i.e., to be equal to the capital plus earnings at 12 percent per year).

INCOME TAXES

Corporate income taxes are levied by the U.S. government and by some of the states as well. The federal income taxes are by far the largest of these. State income taxes are often calculated by closely following federal policies. Therefore, federal corporate taxes are emphasized here. The federal government levies some other taxes as well.

Federal Income Taxes

Because the federal corporate income tax rate is as high as 39 percent of net profit, it is an extremely important component in corporate planning. The actual federal marginal, or incremental, corporate income tax rates, as of September 2000, are shown in Table 7-6, and they range from 15 to 39 percent of taxable income. Average corporate income tax rates are shown in Table 7-7; these range from 15 to 35 percent of taxable income as the income increases. Clearly the tax rate is quite dependent on taxable income. To estimate corporate income tax for a new project, the actual incremental, or marginal, tax rate associated with the taxable income added to corporate earnings should be used, if known. For convenience, a rate of 35 percent will be used here, unless stated otherwise.

Taxable Income

Income taxes are paid on a corporatewide basis. The taxable income of a corporation is the total gross profit. Gross profit, or gross earnings, equals total revenue minus

Table 7-6 U.S. corporate incremental income tax rates†

Taxable income		Incremental tax rate, %
Over	But not over	
$ 0	$ 50,000	15
50,000	75,000	25
75,000	100,000	34
100,000	335,000	39
335,000	10,000,000	34
10,000,000	15,000,000	35
15,000,000	18,333,333	38
18,333,333	—	35

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Table 7-7 U.S. corporate average income tax rates†

Taxable income		Average tax rate, % (increases are linear with income over each range)	
Over	But not over	From	To
$ 0	$ 50,000	15	15
50,000	75,000	15	18.333
75,000	100,000	18.333	22.25
100,000	335,000	22.25	34
335,000	10,000,000	34	34
10,000,000	15,000,000	34	34.333
15,000,000	18,333,333	34.333	35
18,333,333	—	35	35

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total product cost (TPC).[†] Total revenue is income from all sources, primarily product sales, but including sales of assets and supplies, royalties, and other revenues. Dividends and interest paid to the corporation and its shareholders are not considered as allowable costs of doing business for income tax purposes, nor are repayments of loan principal, and therefore cannot be subtracted from revenues in the calculation of gross profit.

In the assessment of the performance of a particular unit or process within a corporation, the revenues and costs associated with that process are determined and used in the evaluation. In addition, there are expenses incurred at levels above that of the individual operating units, at the plant, division, or corporate level. These include such items as safety, payroll, restaurant, recreation, control laboratories, waste disposal, administrative costs, donations, advertising, and research and development. Such costs are allocated to particular processes, usually as a percentage of the capital investment in each process. In evaluating a new investment, the income tax attributable to that investment is the gross income it is expected to generate, multiplied by the marginal income tax rate associated with the addition of that income to corporate income. It is common in evaluations to use a fixed income tax rate, such as the 35 percent noted above.

Corporate income tax payments are due in installments of 25 percent of the projected annual total in mid-April, June, September, and December. In cash flow calculations, they are usually treated in the same manner as other cash flows, that is, as occurring either once a year at year end or continuously.

Tax law is subject to legislative actions, Internal Revenue Service rulings, and court interpretations; so there are frequent changes. Methods for calculating and including taxes in economic evaluations are often established by corporate policy. The engineer charged with conducting evaluations should consult with company tax accounting and legal departments for current policy interpretations and for corporate procedures.[‡]

Capital Gains Tax

A *capital gains tax* is levied on profits made from the sale of capital assets, such as land, buildings, and equipment. The profit on the sale of land equals the selling price less the acquisition price, costs of selling, and costs of improvements. Land is not depreciable. For depreciable assets, such as buildings and equipment, profit is the selling price less the costs of selling, and less the cost of acquisition reduced by the amount of depreciation that has already been charged. Thus, a piece of equipment originally purchased for \$80,000 which has already had \$50,000 of depreciation charged as an expense, and is sold for \$45,000 with \$2000 in selling expense—advertising and removal from service—would show a capital gain of $\$45{,}000 - (80{,}000 - 50{,}000) - 2000 = \$13{,}000$.

[†]Total product cost is summarized in detail in Fig. 6-7 and Table 6-18.

[‡]Complete details are available in *Income Tax Regulations* and periodic *Income Tax Bulletins* issued by the U.S. Treasury, Superintendent of Documents, and Internal Revenue Service. Helpful information may also be found in annual private publications, such as *Prentice-Hall Federal Tax Advisor,* Prentice-Hall Information Services, Paramus, NJ, and *U.S. Master Tax Guide,* CCH, Inc., Chicago.

The capital gain on an item held for 1 year or more before sale is known as a long-term capital gain. The capital gain on items held for less than 1 year is designated as a short-term capital gain. Under current income tax laws, the tax rate on a long-term capital gain is 20 percent, while that on a short-term capital gain is equal to the incremental income tax rate. Normally, a 35 percent rate can be used for estimates.

Losses

The tax laws also make provisions for losses as well. Losses within a company may be used to offset gains within the company in the same year, thereby reducing the taxable income. If total corporate operations show a loss within a given year, that loss can be carried back for up to 3 years to offset past profits; or they may be carried forward for up to 5 years to offset future profits.

A particular project may be profitable overall, but show losses in one or more years. In such cases, the question arises as to whether the losses should be used to reduce overall corporate income taxes or whether these reductions should be used as a "negative income tax" in the evaluation of the process. Even though it may be realistic to do so, it is unwise to use this argument for justifying the economic viability of a project. Here, again, company policy determines the decision.

Other Federal Taxes

Employers pay a percentage of employees' wages as a contribution to Social Security and Medicare taxes. These are considered part of the total employee benefits package which includes medical insurance and retirement plans that are usually combined with wages to obtain labor costs. Such benefits depend upon individual corporate benefits plans, but a typical value is 40 percent of wages.

State Taxes

Some states in the United States levy a corporate income tax, while many do not. Moreover, the tax rate differs among states that do have an income tax. If state income tax is to be included in an economic analysis, the state in which the project is to take place and the corporate taxation policies of that state must be determined and factored into the analysis. Some analysts choose to add a number to the federal income tax rate to reflect the possibility of state taxes; for example, the average federal rate of 35 percent might be increased to 40 percent for preliminary economic analyses. This is another case where specific information or corporate policy should be used to guide the choice.

There are other state taxes as well, for example, workers' compensation taxes. Here again, specific knowledge of the taxation policies at the site of the project is needed to determine the allowances to be made.

Nonincome Taxes

In addition to income taxes, property and excise taxes may be levied by federal, state, or local governments. Local governments usually have jurisdiction over property taxes, which are commonly charged on a county and city basis. Taxes of this type are referred to as *direct* since they must be paid directly by the particular business and cannot be passed on as such to the consumer.

Property taxes may vary widely from one locality to another, but the average annual amount of these charges is 1 to 4 percent of the assessed valuation. The method and the amount of assessment of property value differ considerably with locale as well. For economic evaluation purposes, 2 percent of the fixed-capital investment in a project is a typical property tax rate charge.

Excise taxes are levied by federal and state governments. Federal excise taxes include charges for import customs duties, transfer of stocks and bonds, and a large number of similar items. Manufacturers' and retailers' excise taxes are levied on the sale of many products such as gasoline and alcoholic beverages. Taxes of this type are often referred to as *indirect* since they can be passed on to the consumer. Many businesses must also pay excise taxes for the privilege of carrying on a business or manufacturing enterprise in their particular localities. Excise taxes are included in economic evaluations only when they are known to apply specifically to the project under consideration.

FIXED CHARGES

Among the many costs included in the total product cost, as listed in Fig. 6-7, are a group of *fixed charges*. The first three of these charges—depreciation, property taxes (discussed above), and insurance—are costs related to the capital investment in a project. Once the facility for the project has been built, these charges are fixed. The other fixed charge, rent, is also fixed if there has been a contractual agreement for the rental of space. These charges are fixed in that they are not related to the activity level of the plant and continue even if the plant does not operate. Interest is sometimes included as a fixed charge, but because of its importance has been treated separately in this chapter.

Depreciation

Depreciation is an unusual charge in that it is paid into the corporate treasury. There are other kinds of intracorporate transfers, such as material and utility purchases from one division by another. Such transfers generally have little or no overall impact on the corporate finances. Depreciation, however, has a significant effect on corporate cash flow.

The concept of depreciation is based upon the fact that physical facilities deteriorate and decline in usefulness with time; thus, the value of a facility decreases. *Physical depreciation* is the term given to the measure of the decrease in value of a facility due to changes in the physical aspects of a property. Wear and tear, corrosion, accidents, and deterioration due to age or the elements are all causes of physical depreciation. With this type of depreciation, the serviceability of a property is reduced because of the physical changes.

Depreciation due to all other causes is known as *functional depreciation*. One common type of functional depreciation is obsolescence. This is caused by technological advances which make an existing property obsolete. Other causes of functional depreciation could be (1) decrease in demand for the service rendered by the property, (2) shifts in population, (3) changes in requirements of public authority, (4) inadequacy or insufficient capacity, and (5) abandonment of the enterprise.

Depreciation and Income Tax

Because depreciation impacts the federal income tax liability of a corporation, the federal government, as well as the corporation, has an interest in depreciation. Indeed, federal tax laws closely control the manner in which depreciation is charged.

Depreciation is a charge to the revenue resulting from an investment in real property. It is entirely reasonable that invested principal should be recovered by the investor and that project revenues be charged to pay that principal. In the case of other investments, such as savings accounts, the original investment is available in addition to any return that has been earned, and thus a recovery of invested capital is to be expected in plant investments as well. Depreciation is charged as an expense and then paid to the corporation. It is added and subtracted on the corporate books, and because of this, it is sometimes referred to as an accounting artifact. Depreciation is more than an artifact, however, because of the effect it has on the amount of income tax that a corporation must pay. One definition of *depreciation* is as follows: "A deduction for depreciation may be claimed each year for property with a limited useful life that's used in a trade or business or held for the production of income. This deduction allows taxpayers to recover their costs for the property over a period of years."[†]

Amortization, a word sometimes used interchangeably with depreciation, has a more restricted meaning in tax policy: "You may claim an *amortization* deduction for intangibles with limited useful lives that can be estimated with reasonable accuracy. For example, patents and copyrights are amortizable."[‡]

It has been shown in Chap. 6 that depreciation adds to the corporate treasury as indicated by the following relationship:

$$A_j = (s_j - c_{oj})(1 - \Phi) + \Phi d_j \qquad (6\text{-}1)$$

where the subscript j indicates an annual value in year j, A the annual cash flow, s the annual sales revenue, c_o the operating cost (all of total product cost except depreciation), Φ the fractional income tax rate, and d the annual depreciation. The positive term Φd_j in Eq. (6-1) results from subtracting depreciation as an expense before income taxes are calculated. This reduces the taxable income by d_j, the taxes owed by Φd_j, and the net income by $(1 - \Phi)d_j$. The recovery of the depreciation increases the net cash flow from the project to the capital source of the company (see Fig. 6-1 for further details). This reflects recognition in the tax code that depreciation is the recovery of the original investment.

[†]*1993 Prentice-Hall Federal Tax Advisor,* Prentice-Hall Tax and Professional Practice, Englewood Cliffs, NJ, 1993, chap. 11.

[‡]*1993 Prentice-Hall Federal Tax Advisor,* loc. cit.

Depreciable Investments

In general, all property with a limited useful life of more than 1 year that is used in a trade or business, or held for the production of income, is depreciable. Physical facilities, including such costs as design and engineering, shipping, and field erection, are depreciable. Land is not depreciable, but improvements to the land, such as grading and adding utility services, are depreciable. Working capital and start-up costs are not depreciable. Inventories held for sale are not depreciable. In project terminology, the fixed-capital investment, not including land, is depreciable.

Maintenance is necessary for keeping a property in good condition; repairs connote the mending or replacing of broken or worn parts of a property. The costs of maintenance and repairs are direct operating expenses and thus are not depreciable.

The total amount of depreciation that may be charged is equal to the amount of the original investment in a property—no more and no less. Depreciation does not inflate or deflate.

Current Value

The *current value* of an asset is the value of the asset in its condition at the time of valuation. *Book value* is the difference between the original cost of a property and all the depreciation charged up to a time. It is important, because it is included in the values of all assets of a corporation. The method of determining depreciation may be different for purposes of obtaining the book value than that which is used for income tax purposes, depending on corporate policy. The price that could be obtained for an asset if it were sold on the open market is designated the *market value*. It may be quite different from the book value and clearly is important for determining the true asset value of the company.

Salvage Value

Salvage value is the net amount of money obtainable from the sale of used property over and above any charges involved in removal and sale. The term *salvage value* implies that the property can be of further service. If the property is not useful, it can often be sold for material recovery. Income obtainable from this type of disposal is known as *scrap value*. As of 2002, tax laws do not permit considering salvage or scrap value in the calculation of depreciation. Income from the sale of used property, to the extent that it exceeds the undepreciated value of the property, is therefore taxed as a capital gain. If the net sale price is less than the undepreciated value, it is not taxable but it is included as an income to the project at the time of the sale.

Recovery Period

The period over which the use of a property is economically feasible is known as the *service life* of the property. The period over which depreciation is charged is the *recovery period,* and this is established by tax codes. While originally the recovery period was at least approximately related to the service life, the reality now is that there is little relationship between the two. Recovery periods for some chemical- and process-industries-related depreciation are shown in Table 7-8.

Table 7-8 Recovery periods for selected chemical-industry-related asset classes†

	Recovery period, years	
Type of assets	MACRS	Straight line
Heavy general-purpose trucks	5	5
Industrial steam and electric generation and/or distribution systems	15	22
Information systems (e.g., computers)	5	5
Manufacture of chemicals and allied products (including petrochemicals)	5	9.5
Manufacture of electronic components, products, and systems	5	5
Manufacture of finished plastic products	7	11
Manufacture of other (than grain, sugar, and vegetable oils) food and kindred products	7	12
Manufacture of pulp and paper	7	13
Manufacture of rubber products	7	14
Manufacture of semiconductors	5	5
Petroleum refining	10	16
Pipeline transportation	15	22
Gas utility synthetic natural gas (SNG)	7	14
SNG—coal gasification	10	18
Liquefied natural gas plant	15	22
Waste reduction and resource recovery plant	7	10
Alternative energy property	5	12

†Source: © 2002 CCH Incorporated. All Rights Reserved. Reprinted with permission from *1997 Depreciation Guide Featuring MACRS.*

Methods for Calculating Depreciation

There are several methods for calculating depreciation;† however, because of the federal income tax rules in effect since 1987, we shall consider here only three of the methods: straight-line, double-declining balance, and the modified accelerated cost recovery system (MACRS).

Depreciation results in a reduction in income tax payable in the years in which it is charged. The total amount of depreciation that can be charged is fixed and equal to the investment in depreciable property. Thus, over any recovery period, the same total amount is depreciated; hence, the same total amount of tax is paid—assuming that the incremental tax rate is the same in all those years. However, because money has a time value, it is economically preferable to receive benefits, including tax savings, sooner rather than later. Therefore, it is usually in the taxpayer's interest to depreciate property as rapidly as possible. From the federal government's perspective, however, for

†M. S. Peters and K. D. Timmerhaus, *Plant Design and Economics for Chemical Engineers,* 4th ed., McGraw-Hill, New York, 1991.

the same reason, it is preferable to receive tax revenues sooner rather than later. Counterbalancing this, from the government's point of view, is the desire to encourage business activity and thus the overall economy. For these reasons, the rate and length of time during which depreciation can be charged are a matter of government policy.

Straight-Line Method This method may be elected under the federal tax code as an *alternative depreciation system*. It depreciates property less rapidly than does MACRS, and therefore, it would only be chosen for use in tax computations under special circumstances. For example, a new company might wish to conserve depreciation deductions for use in the future when its incremental tax rate is expected to be higher. For purposes of economic evaluation of projects, straight-line depreciation is often used when employing a profitability measure that does not consider the time value of money, because under these circumstances the rate of depreciation is not important.

In the straight-line method, the property value is assumed to decrease linearly with time over the recovery period. No salvage or scrap value may be taken. Thus the amount of depreciation in each year of the recovery period is

$$d = \frac{V}{n} \tag{7-44}$$

where d is the annual depreciation in dollars per year, V the original investment in the property at the start of the recovery period, and n the length of the straight-line recovery period. For tax purposes, the recovery period for straight-line depreciation is 9.5 years for chemical plants, as shown in Table 7-8. For purposes other than tax calculations, the corporation may select a value for n. If straight-line depreciation is being used in conjunction with a profitability measure that does not take into account the time value of money, then one reasonable recovery period to use is the length of the evaluation period. Another reasonable choice is 6 years, because this gives the same average rate of depreciation as does MACRS.

Modified Accelerated Cost Recovery System MACRS is the depreciation method used for most income tax purposes and therefore also for most economic evaluations. The MACRS method is based upon the classical double-declining-balance method,[†] but with no salvage or scrap value allowed, a switch to straight-line at a point, and use of the half-year convention. There are also mid-month and mid-quarter conventions, but they rarely occur in corporate project tax situations.

The double-declining-balance method allows a depreciation charge in each year of the recovery period that is twice the average rate of recovery on the remaining undepreciated balance for the full recovery period. Thus, in the first year of a 5-year recovery period, the fraction of the original depreciable investment that can be taken as depreciation is $(2)(\frac{1}{5})$, or 40 percent. The undepreciated portion is now 60 percent of the original investment; thus, in the second year, the allowable amount is $(2)(\frac{0.6}{5})$, or

[†]M. S. Peters and K. D. Timmerhaus, *Plant Design and Economics for Chemical Engineers,* 4th ed., McGraw-Hill, New York, 1991, chap. 9.

24 percent; and so on. Because this method always takes a fraction of the remaining balance, the asset is never fully depreciated. The MACRS method overcomes this by employing a shift to the straight-line method in the first year that the straight-line depreciation provides a higher depreciation rate than the declining-balance method.

The *half-year convention* indicates that in the first year only one-half of the double-declining-balance method is allowed and the balance remaining after the end of the recovery period is depreciated in the next year. This leads to the strange result that the MACRS depreciation always requires an additional year over the length of the recovery period.

EXAMPLE 7-9 Calculation of the MACRS Yearly Depreciation Percentage

Calculate the annual percentage rate of depreciation for a 5-year recovery period asset, such as a chemical plant, using the double-declining-balance method and the half-year convention, and switching to the straight-line method on the remaining balance when it gives a higher annual depreciation than that obtained with the double-declining-balance method. This is the MACRS method.

■ Solution

First year: The double-declining-balance (DDB) method allows a deprecation of $(2)(\frac{1}{5}) = 0.4$, but the half-year convention reduces this by one-half to 0.20, or 20 percent.

The straight-line (SL) method for the first year permits a depreciation of $\frac{1}{5} = 0.20$, or 20 percent, the same as the DDB method with the half-year convention.

Second year: The undepreciated balance is now $1 - 0.2 = 0.8$ of the original. The DDB method allows a depreciation of $(2)(0.8/5) = 0.32$, or 32 percent. The SL method, with 4.5 years remaining, allows $0.8/4.5 = 0.177$, so the DDB method should be used.

Third year: The undepreciated balance is now $1 - 0.52 = 0.48$ of the original. The DDB method allows a depreciation of $(2)(0.48/5) = 0.192$, or 19.2 percent. The SL method, with 3.5 years remaining, allows $0.48/3.5 = 0.137$; so again, the DDB method should be used.

Fourth year: The undepreciated balance is now $1 - 0.712 = 0.288$. The DDB method allows a depreciation of $(2)(0.288/5) = 0.1152$, or 11.52 percent. The SL method, with 2.5 years remaining, shows a depreciation of $0.288/2.5 = 0.1152$. Both methods give the same value.

Fifth year: The undepreciated balance is now $1 - 0.8272 = 0.1728$. The DDB method allows a depreciation of $(2)(0.1728/5) = 0.06912$, or 6.192 percent. The SL method, with 1.5 years remaining, allows a depreciation of $0.1728/1.5 = 0.1152$, or 11.52 percent. This time the SL method provides the higher rate, and that value is used.

Sixth year: Because of the half-year convention, there is a depreciation charge left for this year. It is the remaining undepreciated balance, amounting to $1 - 0.9424 = 0.0576$, or 5.76 percent. ■

Table 7-9 shows the MACRS annual rates for the half-year convention for all the allowed recovery periods up to 20 years, the half-year convention, and switching to straight line. The 3-, 5-, 7- and 10-year recovery periods employ the double-, or 200 percent, declining-balance method, whereas the 15- and 20-year recovery periods use the 150 percent declining-balance method. U.S. tax law prescribes all these conditions.

Table 7-9 MACRS depreciation rates†

General depreciation system
Applicable depreciation method: 200 or 150 percent
Declining balance switching to straight-line method
Applicable recovery periods: 3, 5, 7, 10, 15, 20 years
Applicable convention: half-year

Recovery year	Recovery period					
	3-year	5-year	7-year	10-year	15-year	20-year
	Depreciation rate, %					
1	33.33	20.00	14.29	10.00	5.00	3.750
2	44.45	32.00	24.49	18.00	9.50	7.219
3	14.81	19.20	17.49	14.40	8.55	6.677
4	7.41	11.52	12.49	11.52	7.70	6.177
5		11.52	8.93	9.22	6.93	5.713
6		5.76	8.92	7.37	6.23	5.285
7			8.93	6.55	5.90	4.888
8			4.46	6.55	5.90	4.522
9				6.56	5.91	4.462
10				6.55	5.90	4.461
11				3.28	5.91	4.462
12					5.90	4.461
13					5.91	4.462
14					5.90	4.461
15					5.91	4.462
16					2.95	4.461
17						4.462
18						4.461
19						4.462
20						4.461
21						2.231

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Insurance

The annual insurance costs for ordinary industrial projects are approximately 1 percent of the fixed-capital investment. Despite the fact that insurance costs may represent only a small fraction of total costs, it is necessary to consider insurance requirements carefully to make certain the economic operation of a plant is protected against emergencies or unforeseen developments.

The design engineer can aid in reducing insurance requirements if all the factors involved in obtaining adequate insurance are understood. In particular, the engineer should be aware of the different types of insurance available and the legal responsibilities of a corporation with regard to accidents and other unpredictable emergencies.

Legal Responsibility A corporation can obtain insurance to protect itself against loss of property due to any of a number of different causes. Protection against unforeseen emergencies other than direct property loss can also be obtained through insurance.

For example, injuries to employees or others due to a fire or explosion can be covered. It is impossible to insure against every possible incidence, but it is necessary to consider the results of a potential emergency and understand the legal responsibility for various types of events. The payments required for settling a case in which legal responsibility has been proved can be much greater than any costs due to direct property damage.

The design engineer should be familiar with laws and regulations governing the type of plant or process involved in a design. In case of an accident, failure to comply with the laws involved is a major factor in establishing legal responsibility. Compliance with all existing laws, however, is not a sufficient basis for disallowance of legal liability. Every known safety feature should be included, and extraordinary care in the complete operation must be proved before a good case can be presented for disallowing legal liability.

Liability for product safety has become a major concern for manufacturers in recent years, due to heightened public awareness of producer's liability. Product testing and hazard warnings are minimal activities to undertake before releasing a product for distribution.

Types of Insurance Many different types of insurance are available for protection against property loss or charges based on legal liability. Despite every precaution, there is always the possibility of an unforeseen event causing a sudden drain on a corporation's finances, and an efficient management protects itself against such potential emergencies by taking out insurance to cover such risks.

The major insurance requirements for manufacturing concerns can be classified as follows:

1. Fire insurance and similar emergency coverage on buildings, equipment, and all other owned, used or stored property. Included in this category would be losses caused by lightning, wind, hailstorms, floods, automobile accidents, explosions, earthquakes, and similar occurrences.
2. Public-liability insurance, including bodily injury and property loss or damage, on all operations such as those involving automobiles, elevators, attractive nuisances, aviation products, or any corporate function carried out at a location away from the plant premises.
3. Business-interruption insurance. The loss of income due to a business interruption caused by fire or other emergency may far exceed any loss in property. Consequently, insurance against a business interruption of this type is extremely important.
4. Power plant, machinery, and special-operations hazards.
5. Workers' compensation insurance.
6. Marine and transportation insurance for all property in transit.
7. Comprehensive crime coverage.
8. Employee-benefit insurance, including life, hospitalization, accident, health, personal property, and pension plans.
9. Product liability.

Self-Insurance

Because the payout of claims by insurance companies is perhaps only 55 to 60 percent for each dollar of premium they receive, self-insurance is sometimes used to minimize the cost of insurance. The decision whether to purchase or self-insure requires balancing the possible savings versus the chances of large losses. The tax implications must be considered as well, because insurance premiums for standard insurance are tax-deductible while funds paid into a self-insurance reserve ordinarily are not. Overall corporate policies dictate the type and amount of insurance that will be held. It should be realized, however, that a well-designed insurance plan needs input from persons who understand all the aspects of insurance as well as the problems involved in the manufacturing operation.

NOMENCLATURE

A = uniform series; series of constant, year-end cash flows, dollars
A_j = cash flow in year j, dollars
$\bar{A}$ = continuous cash flow for N years at a constant rate, dollars
c_{oj} = outflow cost of operation in year j, including all expenses except depreciation, dollars
d = amount of straight-line depreciation, dollars
d_j = depreciation in year j, dollars
F = future amount or future worth of money, dollars
i = interest rate based on length of one interest period, percent/100
i_{eff} = effective interest rate defined as the interest rate with annual compounding which provides the same amount of interest in 1 year as that earned in 1 year with another compounding period, percent/100
I = amount of interest earned in a period, dollars per period
I_j = amount of interest paid in period j, dollars per period
j = specified interest period, dimension of time
L = constant amount of loan repayment per period, dollars per period
m = number of interest periods per year, dimensionless
N = total number of interest periods, dimensionless
n = recovery period, straight-line depreciation, years
p_j = amount of principal repaid in period j, dollars per period
p_m = amount of principal repaid in period m, dollars per period
P = principal or present worth, dollars
P_{j-1} = amount of unpaid principal remaining after $j-1$ loan payments, dollars
P_0 = initial amount of a loan, dollars
$\bar{P}$ = continuous, constant cash flow rate for 1 year, dollars/yr
r = nominal interest rate—for compounding other than annual, percent/100
s_j = amount of sales in year j, dollars
V = amount of depreciable capital investment, dollars