minimum profits as an expense is acceptable providing the base, or minimum return and the general method employed are clearly indicated.

Example 1 Determination of rate of return on investment—consideration of income-tax effects. A proposed manufacturing plant requires an initial fixed-capital investment of $900,000 and $100,000 of working capital. It is estimated that the annual income will be $800,000 and the annual expenses including depreciation will be $520,000 before income taxes. A minimum annual return of 15 percent before income taxes is required before the investment will be worthwhile. Income taxes amount to 34 percent of all pre-tax profits.

Determine the following:

(*a*) The annual percent return on the total initial investment before income taxes.
(*b*) The annual percent return on the total initial investment after income taxes.
(*c*) The annual percent return on the total initial investment before income taxes based on capital recovery with minimum profit.
(*d*) The annual percent return on the average investment before income taxes assuming straight-line depreciation and zero salvage value.

Solution

(*a*) Annual profit before income taxes = $800,000 − $520,000 = $280,000.
Annual percent return on the total initial investment before income taxes = [280,000/(900,000 + 100,000)](100) = 28 percent.

(*b*) Annual profit after income taxes = ($280,000)(0.66) = $184,800.
Annual percent return on the total initial investment after income taxes = [184,800/(900,000 + 100,000)](100) = 18.5 percent.

(*c*) Minimum profit required per year before income taxes = ($900,000 + $100,000)(0.15) = $150,000.
Fictitious expenses based on capital recovery with minimum profit = $520,000 + $150,000 = $670,000/year. Annual percent return on the total investment based on capital recovery with minimum annual rate of return of 15 percent before income taxes = [(800,000 − 670,000)/(900,000 + 100,000)](100) = 13 percent.

(*d*) Average investment assuming straight-line depreciation and zero salvage value = $900,000/2 + $100,000 = $550,000.
Annual percent return on average investment before income taxes = (280,000/550,000)(100) = 51 percent.

The methods for determining rate of return, as presented in the preceding sections, give "point values" which are either applicable for one particular year or for some sort of "average" year. They do not consider the time value of money, and they do not account for the fact that profits and costs may vary significantly over the life of the project.

One example of a cost that can vary during the life of a project is depreciation cost. If straight-line depreciation is used, this cost will remain constant; however, it may be advantageous to employ a declining-balance or sum-of-the-years-digits method to determine depreciation costs, which will immediately result in variations in costs and profits from one year to another. Other predictable factors, such as increasing maintenance costs or changing sales volume, may also make it necessary to estimate year-by-year profits with

variation during the life of the project. For these situations, analyses of project profitability cannot be made on the basis of one point on a flat time-versus-earning curve, and profitability analyses based on discounted cash flow may be appropriate. Similarly, time-value-of-money considerations may make the discounted-cash-flow approach desirable when annual profits are constant.

DISCOUNTED CASH FLOW

Rate of Return Based on Discounted Cash Flow†

The method of approach for a profitability evaluation by discounted cash flow takes into account the time value of money and is based on the amount of the investment that is unreturned at the end of each year during the estimated life of the project. A trial-and-error procedure is used to establish a rate of return which can be applied to yearly cash flow so that the original investment is reduced to zero (or to salvage and land value plus working-capital investment) during the project life. Thus, the rate of return by this method is equivalent to the maximum interest rate (normally, after taxes) at which money could be borrowed to finance the project under conditions where the net cash flow to the project over its life would be just sufficient to pay all principal and interest accumulated on the outstanding principal.

To illustrate the basic principles involved in discounted-cash-flow calculations and the meaning of rate of return based on discounted cash flow, consider the case of a proposed project for which the following data apply:

Initial fixed-capital investment = $100,000
Working-capital investment = $10,000
Service life = 5 years
Salvage value at end of service life = $10,000

Year	Predicted after-tax cash flow to project based on total income minus all costs except depreciation, $ (expressed as end-of-year situation)
0	(110,000)
1	30,000
2	31,000
3	36,000
4	40,000
5	43,000

†Common names of methods of return calculations related to the discounted-cash-flow approach are *profitability index*, *interest rate of return*, *true rate of return*, and *investor's rate of return*.

TABLE 1
Computation of discounted-cash-flow rate of return

Year (n')	Estimated cash flow to project, \$	Trial for $i = 0.15$: Discount factor, $\frac{1}{(1+i)^{n'}}$	Trial for $i = 0.15$: Present value, \$	Trial for $i = 0.20$: Discount factor, $\frac{1}{(1+i)^{n'}}$	Trial for $i = 0.20$: Present value, \$	Trial for $i = 0.25$: Discount factor, $\frac{1}{(1+i)^{n'}}$	Trial for $i = 0.25$: Present value, \$	Trial for $i = 0.207$†: Discount factor, $\frac{1}{(1+i)^{n'}}$	Trial for $i = 0.207$†: Present value, \$
0	(110,000)								
1	30,000	0.8696	26,100	0.8333	25,000	0.8000	24,000	0.829	24,900
2	31,000	0.7561	23,400	0.6944	21,500	0.6400	19,800	0.687	21,200
3	36,000	0.6575	23,300	0.5787	20,700	0.5120	18,400	0.570	20,500
4	40,000	0.5718	22,900	0.4823	19,300	0.4096	16,400	0.472	18,800
5	43,000 +20,000	0.4971	31,300	0.4019	25,300	0.3277	20,600	0.391	24,600
	Total		127,000		111,800		99,200		110,000
Ratio = $\frac{\text{total present value}}{\text{initial investment}}$			1.155		1.016		0.902		1.000
									Trial is satisfactory

†As illustrated in Fig. 10-2, interpolation to determine the correct rate of return can be accomplished by plotting the ratio (total present value/initial investment) versus the trial interest rate for three bracketing values and reading the correct rate from the curve where the ratio = 1.0.

NOTE: *In this example, interest was compounded annually on an end-of-year basis and continuous interest compounding was ignored. Also, construction period and land value were not considered. The preceding effects could have been included in the analysis for a more sophisticated treatment using the methods presented in Examples 2 and 3 of this chapter.*

Designate the discounted-cash-flow rate of return as i. This rate of return represents the after-tax interest rate at which the investment is repaid by proceeds from the project. It is also the maximum after-tax interest rate at which funds could be borrowed for the investment and just break even at the end of the service life.

At the end of five years, the cash flow to the project, compounded on the basis of end-of-year income, will be

$$(\$30{,}000)(1+i)^4 + (\$31{,}000)(1+i)^3 + (\$36{,}000)(1+i)^2 + (\$40{,}000)(1+i) + \$43{,}000 = S \quad (1)$$

The symbol S represents the future worth of the proceeds to the project and must just equal the future worth of the initial investment compounded at an interest rate i corrected for salvage value and working capital. Thus,

$$S = (\$110{,}000)(1+i)^5 - \$10{,}000 - \$10{,}000 \quad (2)$$

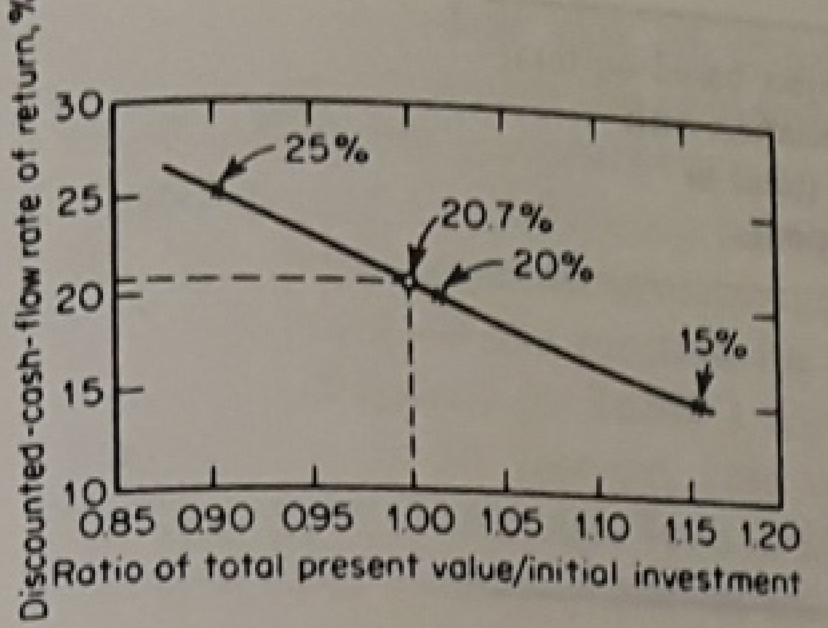

FIGURE 10-2 Graphical analysis for trial-and-error determination of discounted-cash-flow rate of return (see Table 1).

Setting Eq. (1) equal to Eq. (2) and solving by trial and error for i gives $i = 0.207$, or the discounted-cash-flow rate of return is 20.7 percent.

Some of the tedious and time-consuming calculations can be eliminated by applying a *discount factor* to the annual cash flows and summing to get a present value equal to the required investment. The discount factor for end-of-year payments and annual compounding is

$$d_{n'} = \frac{1}{(1+i)^{n'}} = \text{discount factor} \quad (3)$$

where i = rate of return

n' = year of project life to which cash flow applies

This discount factor, $d_{n'}$, is the amount that would yield one dollar after n' years if invested at an interest rate of i. The discounted-cash-flow rate of return can be determined by the trial-and-error method illustrated in Table 1, where the annual cash flows are discounted by the appropriate discount factor to a total present value equal to the necessary initial investment.†

Example 2 Discounted-cash-flow calculations based on continuous interest compounding and continuous cash flow. Using the discount factors for continuous interest and continuous cash flow presented in Tables 5 to 8 of Chapter 7, determine the continuous discounted-cash-flow rate of return r for the example presented in the preceding section where yearly cash flow is continuous. The data follow.

Initial fixed-capital investment = \$100,000
Working-capital investment = \$10,000
Service life = 5 years
Salvage value at end of service life = \$10,000

†The significance of the use of discount factors, as illustrated in Table 1 and Example 2, can be seen by dividing both sides of Eq. (1) and Eq. (2) by $(1+i)^5$, or by $(1+i)^n$ for the general case where n is the estimated service life in years.

TABLE 4
Definitions to clarify income-tax situation for profitability evaluation

Revenue = total income (or total savings)
Net profits = revenue − all expenses − income tax
All expenses = cash expenses + depreciation
Income tax = (revenue − all expenses)(tax rate)
Cash flow = net profits + depreciation
Cash flow = (revenue)(1 − tax rate) − (cash expenses)(1 − tax rate) + (depreciation)(tax rate)
Cash flow = (revenue)(1 − tax rate) − (all expenses)(1 − tax rate) + depreciation

For the case of a 34% tax rate

$1.00 of revenue (either as sales income or savings) yields a cash flow of $0.66.
$1.00 of cash expenses (as raw materials, labor, etc.) yields a cash outflow of $0.66.
$1.00 of depreciation yields a cash inflow of $0.34.

Consideration of Income Taxes

Income-tax effects can be included properly in all the profitability methods discussed in this chapter by using appropriate definitions of terms, such as those presented in Table 4. The methods of discounted-cash-flow rate of return and present worth are limited to consideration of cash income and cash outgo over the life of the project. Thus, depreciation, as a cost, does not enter directly into the calculations except as it may affect income taxes.

Net cash flow represents the difference between all cash revenues and all cash expenses with taxes included as a cash expense. Thus, discounted-cash-flow rate of return and present worth should be calculated on an after-tax basis, unless there is some particular reason for a pretax basis, such as comparison to a special alternate which is presented on a pre-tax basis.

Example 5 Comparison of alternative investments by different profitability methods. A company has three alternative investments which are being considered. Because all three investments are for the same type of unit and yield the same service, only one of the investments can be accepted. The risk factors are the same for all three cases. Company policies, based on the current economic situation, dictate that a minimum annual return on the original investment of 15 percent after taxes must be predicted for any unnecessary investment with interest on investment not included as a cost. (This may be assumed to mean that other equally sound investments yielding a 15 percent return after taxes are available.) Company policies also dictate that, where applicable, straight-line depreciation is used and, for time-value of money interpretations, end-of-year cost and profit analysis is used. Land value and prestartup costs can be ignored.

Given the following data, determine which investment, if any, should be made by alternative-analysis profitability-evaluation methods of

(*a*) Rate of return on initial investment
(*b*) Minimum payout period with no interest charge
(*c*) Discounted cash flow
(*d*) Net present worth
(*e*) Capitalized costs

Investment number	Total initial fixed-capital investment, $	Working-capital investment, $	Salvage value at end of service life, $	Service life, years	Annual cash flow to project after taxes,† $	Annual cash expenses‡ (constant for each year), $
1	100,000	10,000	10,000	5	See yearly tabulation§	44,000
2	170,000	10,000	15,000	7	52,000 (constant)	28,000
3	210,000	15,000	20,000	8	59,000 (constant)	21,000

† This is total annual income or revenue minus all costs except depreciation and interest cost for investment.
‡ This is annual cost for operation, maintenance, taxes, and insurance. Equals total annual income minus annual cash flow.
§ For investment number 1, variable annual cash flow to project is: year 1 = $30,000, year 2 = $31,000, year 3 = $36,000, year 4 = $40,000, year 5 = $43,000.

Solution

(*a*) **Method of rate of return on initial investment.**

Average annual profit = annual cash flow − annual depreciation cost

The average annual profits for investment No. 1, using straight-line depreciation are as follows:

Year	Average annual profit, dollars
1	$30{,}000 - \dfrac{(100{,}000 - 10{,}000)}{5} = 30{,}000 - 18{,}000 = 12{,}000$
2	31,000 − 18,000 = 13,000
3	36,000 − 18,000 = 18,000
4	40,000 − 18,000 = 22,000
5	43,000 − 18,000 = 25,000
Total	90,000

For investment No. 1, the arithmetic average of the annual profits is 90,000/5 = $18,000.¶

The annual average rate of return on the first investment is

$$\frac{18{,}000}{100{,}000 + 10{,}000}(100) = 16.4\% \text{ after taxes}$$

¶An alternate method to obtain the average of the annual profits would be to determine the amount of the annuity R based on the end-of-year payments that would compound to the same future worth as the individual profits using an interest rate i of 0.15. With this approach, the average of the annual profits for investment No. 1 would be $17,100.

The method for determining this $17,100 is to apply the series compound-amount factor $[(1 + i)^n - 1]/i$ [see Eq. (21) in Chap. 7] to the annuity to give the future worth S of the annual incomes. The expression is $(12{,}000)(1 + i)^4 + (13{,}000)(1 + i)^3 + (18{,}000)(1 + i)^2 + (22{,}000)(1 + i) + 25{,}000 = R[(1 + i)^5 - 1]/i$. Solving for the case of $i = 0.15$ gives $R = \$17{,}100$.

Because this return is greater than 15 percent, one of the three investments will be recommended, and it is only necessary to compare the three investments.

For investment number	Total initial investment	Average annual profit, dollars
1	\$110,000	\$18,000
2	\$180,000	$52{,}000 - \dfrac{170{,}000 - 15{,}000}{7} = 52{,}000 - 22{,}100 = \$29{,}900$
3	\$225,000	$59{,}000 - \dfrac{210{,}000 - 20{,}000}{8} = 59{,}000 - 23{,}800 = \$35{,}200$

Comparing investment No. 2 with investment No. 1,

$$\text{Percent return} = \frac{29{,}900 - 18{,}000}{180{,}000 - 110{,}000}(100) = 17.0\%$$

Therefore, investment No. 2 is preferred over investment No. 1. Comparing investment No. 3 with investment No. 2,

$$\text{Percent return} = \frac{35{,}200 - 29{,}900}{225{,}000 - 180{,}000}(100) = 11.8\%$$

This return is not acceptable, and *investment No. 2 should be recommended.*

The same result would have been obtained if a minimum return of 15 percent had been incorporated as an expense.

(*b*) **Method of minimum payout period with no interest charge.**

$$\text{Payout period (with no interest charge)} = \frac{\text{depreciable fixed-capital investment}}{\text{avg profit/yr + avg depreciation/yr}}$$

For investment number	*Payout period, years*
1	$\dfrac{90{,}000}{18{,}000 + 18{,}000} = 2.50$
2	$\dfrac{155{,}000}{29{,}900 + 22{,}100} = 2.98$
3	$\dfrac{190{,}000}{35{,}200 + 23{,}800} = 3.22$

The payout period for investment No. 1 is least; therefore, by this method, *investment No. 1 should be recommended.*

(*c*) **Method of discounted cash flow.** For investment No. 1, as illustrated in Table 1, the rate of return based on discounted cash flow is 20.7 percent.

For investment No. 2, the discounted-cash-flow equation is

$$(52{,}000)\left[\frac{1}{1+i} + \frac{1}{(1+i)^2} + \cdots + \frac{1}{(1+i)^7}\right] + (10{,}000 + 15{,}000)\frac{1}{(1+i)^7}$$
$$= \$180{,}000$$

By trial-and-error solution, the discounted-cash-flow rate of return is 22.9 percent

Similarly, for investment No. 3,

$$(59{,}000)\left[\frac{1}{1+i} + \frac{1}{(1+i)^2} + \cdots + \frac{1}{(1+i)^8}\right] + (15{,}000 + 20{,}000)\frac{1}{(1+i)^8}$$
$$= \$225{,}000$$

By trial-and-error solution, the discounted-cash-flow rate of return is 21.5 percent.

To make a choice among the three alternatives, it is necessary to make a comparison among the three possible choices. This comparison can be made in a relatively straightforward manner using discounted-cash-flow rates of return by comparing pairs of investments on a mutually exclusive basis if the various alternatives have the same economic service lives. When different lengths of service life are involved, as in this problem, the best approach is to avoid the calculated rates of return and make the investment comparison by the net present-worth method as shown in part (*d*) of this problem. It would be possible to use discounted-cash-flow rates of return for comparison between investments with different service lives by assuming that each investment could be repeated at the end of its service life until a common end point was obtained for the investments being compared; however, this method becomes very involved mathematically and is not realistic.

If the service lives of the investments being compared are not widely different, the following *approximate* method using discounted-cash-flow rate of return can be employed for the comparison.†

In comparing a pair of alternatives, the base time is chosen as the longer of the two service lines. For the case of the investment with the shorter life, it is assumed that the accumulated amount at the end of its life can be invested at the minimum acceptable rate for the remaining time to equalize the two lives. The rate of return on the incremental investment can then be determined.

Comparison of investment No. 2 to investment No. 1. At the end of its 7-year service life, the net value of investment No. 2 is

$$(180{,}000)(1 + 0.229)^7 + 10{,}000 + 15{,}000 = \$785{,}000$$

With investment No. 1, the net value after 7 years is the amount accumulated in 5 years times the factor to let this accumulated amount be invested at 15 percent for 2 more years, or

$$\left[(110{,}000)(1 + 0.207)^5 + 10{,}000 + 10{,}000\right](1 + 0.15)^2 = \$398{,}000$$

Therefore, a gain of \$785,000 − \$398,000 = \$387,000 is made in 7 years by an added investment of \$70,000 if investment No. 2 is made instead of investment No. 1. The discounted-cash-flow rate of return for this incremental investment is found by

$$(70{,}000)(1 + i)^7 = 387{,}000$$
$$i = 0.277 \quad \text{or} \quad 27.7\%$$

This return is greater than 15 percent; so investment No. 2 is preferred over investment No. 1.

†The method is shown to illustrate the use of discounted-cash-flow rates of return for investment comparisons. It is correct only for comparisons involving equal service lives. If service lives are different, this method tends to favor the investment with the longest service life.

Comparison of investment No. 3 to investment No. 2. At the end of its 8-year service life, the net value of investment No. 3 is

$$(225{,}000)(1 + 0.215)^8 + 15{,}000 + 20{,}000 = \$1{,}105{,}000$$

For comparison, \$180,000 invested in investment No. 2 would, with the last year at a 15 percent return, accumulate in 8 years to

$$\left[(180{,}000)(1 + 0.229)^7 + 10{,}000 + 15{,}000\right](1 + 0.15) = \$903{,}000$$

Therefore, a gain of \$1,105,000 − \$903,000 = \$202,000 is made in 8 years by an added investment of \$45,000 by making investment No. 3 instead of investment No. 2. The discounted-cash-flow rate of return for this incremental investment is found by

$$(45{,}000)(1 + i)^8 = 202{,}000$$

$$i = 0.208 \quad \text{or} \quad 20.8\%$$

This return is greater than 15 percent; so investment No. 3 is preferred over investment No. 2. Therefore, *investment No. 3 should be recommended.*

(*d*) **Method of net present worth.** For investment No. 1, as illustrated in Table 1, the present value of the cash flow to the project, discounted at an interest rate of 15 percent, is \$127,000. Therefore, the net present worth of investment No. 1 is \$127,000 − \$110,000 = \$17,000.

For investments 2 and 3, the present values of the cash flows to the projects are determined from the first two equations under part (*c*) of this problem, with $i = 0.15$. The resulting net present worth are:

For investment No. 2, net present worth = \$226,000 − \$180,000 = \$46,000

For investment No. 3, net present worth = \$278,000 − \$225,000 = \$53,000

The greatest net present worth is found for investment No. 3; therefore, *investment No. 3 should be recommended.*

(*e*) **Method of capitalized costs.** Capitalized costs for each investment situation must include the capitalized cost for the original investment to permit an indefinite number of replacements plus the capitalized present value of the cash expenses plus working capital.

Capitalized present value of cash expenses is determined as follows:

Let $C_{n'}$ be the annual cash expense in year n' of the project life. The present value of the annual cash expenses is then

$$\sum_{n'=1}^{n'=n} C_{n'} \frac{1}{(1+i)^{n'}}$$

and the capitalized present value is

$$\frac{(1+i)^n}{(1+i)^n - 1} \sum_{n'=1}^{n'=n} C_{n'} \frac{1}{(1+i)^{n'}}$$

If $C_{n'}$ is constant, as is the case for this example, the capitalized present value becomes (annual cash expenses)/i. Therefore,

$$\text{Capitalized cost} = \frac{C_R(1+i)^n}{(1+i)^n - 1} + V_s + \frac{\text{annual cash expenses}}{i} + \text{working capital}$$

where n = service life
i = annual rate of return
C_R = replacement cost
V_s = salvage value

For investment No. 1,

$$\text{Capitalized cost} = \frac{(90{,}000)(1 + 0.15)^5}{(1 + 0.15)^5 - 1} + 10{,}000 + \frac{44{,}000}{0.15} + 10{,}000 = \$492{,}000$$

For investment No. 2,

$$\text{Capitalized cost} = \frac{(155{,}000)(1 + 0.15)^7}{(1 + 0.15)^7 - 1} + 15{,}000 + \frac{28{,}000}{0.15} + 10{,}000 = \$460{,}000$$

For investment No. 3,

$$\text{Capitalized cost} = \frac{(190{,}000)(1 + 0.15)^8}{(1 + 0.15)^8 - 1} + 20{,}000 + \frac{21{,}000}{0.15} + 15{,}000 = \$457{,}000$$

The capitalized cost based on a minimum rate of return of 15 percent is least for investment No. 3; therefore, *investment No. 3 should be recommended.*

Note: Methods (*a*) and (*b*) in this problem give incorrect results because the time value of money has not been included. Although investment No. 3 is recommended by methods (*c*), (*d*), and (*e*), it is a relatively narrow choice over investment No. 2. Consequently, for a more accurate evaluation, it would appear that the company management should be informed that certain of their policies relative to profitability evaluation are somewhat old fashioned and do not permit the presentation of a totally realistic situation. For example, the straight-line depreciation method may not be the best choice, and a more realistic depreciation method may be appropriate. The policy of basing time-value-of-money interpretations on end-of-year costs and profits is a simplification, and it may be better to permit the use of continuous interest compounding and continuous cash flow where appropriate. For a final detailed analysis involving a complete plant, variations in prestartup costs among alternatives may be important, and this factor should not be ignored.

REPLACEMENTS

The term "replacement," as used in this chapter, refers to a special type of alternative in which facilities are currently in existence and it may be desirable to replace these facilities with different ones. Although intangible factors may